

BY USING DESIGN THINKING CONCEPT ON COMPARATIVE STUDY OF MAHGOUB AND ELZAKI TRANSFORMS

I.Rajeswari¹ C.Saranya² Dr.A.Viswanathan³

¹Assistant Professor, Department of Mathematics, SNS College of Technology, Coimbatore, Tamilnadu, India.

²Assistant Professor, Department of Mathematics, SNS College of Technology, Coimbatore, Tamilnadu, India.

³Professor and Dean /S&H, Department of Mathematics, SNS College of Technology, Coimbatore, Tamilnadu, India.

Abstract

Design thinking is generally defined as an analytic and creative process that engages a person in opportunities to experiment, create and prototype models, gather feedback, and redesign. Many advanced problems, which appear in the field of engineering and sciences like heat conduction problems, mechanical oscillation problems, vibrating beams problems, electric circuit problems, population growth and radioactive decay problems, can be solved by integral transforms. In this paper, we present a comparative study of two integral transforms namely Mahgoub and Elzaki transforms and solve some systems of differential equations (Homogeneous & Non-Homogeneous) using both the transforms in application section. Results show that Mahgoub and Elzaki transforms are closely connected. The primary purpose of this paper is to apply the design thinking concept to the integral transforms.

Keywords: Design thinking, Mahgoub transform, Elzaki transform, System of differential equations.

I. Introduction

Design thinking is a blend of logic, powerful imagination, systematic reasoning and intuition to bring to the table the ideas that promise to solve the problems of the clients with desirable outcomes. It helps to bring creativity with business insights. Design thinking is a non-linear, iterative process that teams use to understand users, challenge assumptions, redefine problems and create innovative solutions to prototype and test. Involving five phases—Empathize, Define, Ideate, Prototype and Test—it is most useful to tackle problems that are ill-defined or unknown. With design thinking, teams have the freedom to generate ground-breaking solutions.

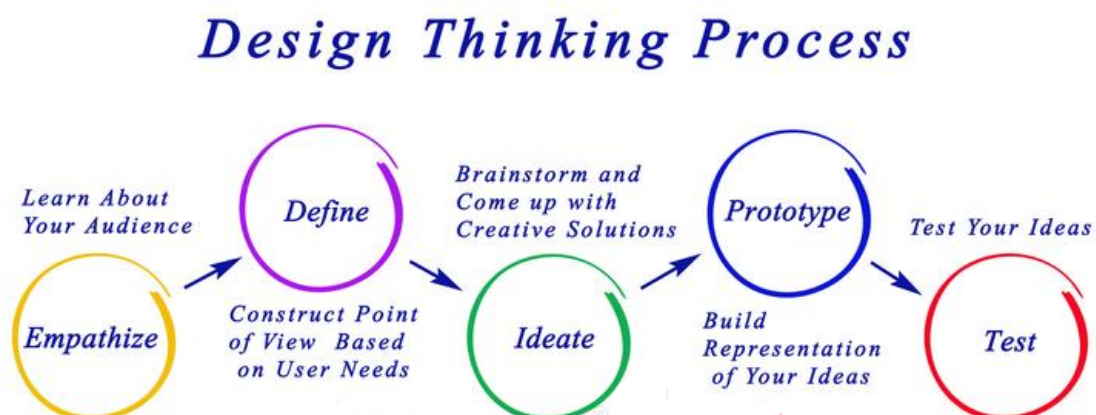


Fig 1. Design Thinking Process

Application of Mahgoub transform for solving linear Volterra integral equations of first kind was given by Aggarwal . He apply Mahgoub transform for solving population growth and decay problems. Kiwne and Sonawane defined fundamental properties of Mahgoub transform with applications. He gave Mahgoub transform of Bessel's functions. He used Mahgoub transform for solving linear ordinary differential equations with variable coefficients. ELzaki Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the Elzaki transform and its fundamental properties. Elzaki transform was introduced by Tarig ELzaki to facilitate the process of solving ordinary and partial differential equations in the time domain. Typically, Fourier, Laplace and Sumudu transforms are the convenient mathematical tools for solving differential equations. Also Elzaki transform and some of its fundamental properties are used to solve differential equations.

II . Emphathy

Polynomials, Algebra, Probability, Integrations, and Differentiations etc.. forms a significant part of the tools used to solve the systems. With the increasing complexity of systems, very sophisticated methods are required. Differential equations are prominently used for defining control systems. These equations are simple to solve. But complexity arises while solving higher order differential equations. To solve such complex higher order differential equations, the mathematical method that proved to be effective is the integral Transforms . As this transform is widely employed, it is useful to know what they really meant for and how do they work.

III . Define

In mathematics, transforms are applied for transforming a variable from one form to another to make the equation easy to handle. The integral transforms for higher order differential equation into a polynomial form which is far easy than solving differential equation directly. The major advantage of the integral transform is that, they are defined for both stable and unstable systems.

- To determine if a function is of exponential order or not.
- To express some simple functions in terms of unit step and/or unit impulse
- To know initial-value theorem and final value theorem and how it can be used
- To perform algebraic manipulation of complex numbers.
- To know integral transform of integral and derivatives (first and high orders derivatives).
- To obtain inverse transforms of simple function using the Table of integral transform pairs.
- To use the method of partial fraction expansion to express strictly proper functions as the sum of simple factors
- To perform long division and know the reason for using it in inverse integral transform.
- To solve constant coefficient linear ordinary differential equations using integral transform.
- To derive the integral transform of time-delayed functions

IV . Ideate

Integral transforms methods (Laplace transform , Fourier transform , Mahgoub transform , Kamal transform , Aboodh transform , Mohand transform , Elzaki transform , Shehu transform , Sumudu transform and Sadik transform) are convenient mathematical tools for solving advance problems of sciences and engineering which are expressible in terms of differential equations, delay differential equations, system of differential equations, partial differential equations, integral equations, system of integral equations, partial integro-differential equations and integro differential equations.

In this paper, we concentrate mainly on the comparative study of Mahgoub and Elzaki transforms and we solve some systems of differential equations using these transforms using design thinking concepts.

V. Prototype

Definition of Mahgoub And Elzaki Transforms

5.1 Definition of Mahgoub transforms:

Mahgoub transform of the function $F(t)$ for all $t \geq 0$ is defined as

$$M\{f(t)\} = v \int_0^{\infty} F(t) e^{-vt} dt = f(v)$$

Where the operator M is called the Mohand transform operator.

5.2 Definition of Elzaki transforms:

Elzaki transform of the function $F(t)$ for all $t \geq 0$ is defined as

$$E\{f(t)\} = v \int_0^{\infty} F(t) e^{-\frac{t}{v}} dt = T(v)$$

Where the operator E is called the Elzaki transform operator.

The Mahgoub and Elzaki transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mahgoub and Elzaki transforms of the function $F(t)$.

Properties Of Mahgoub And Elzaki Transforms

In this section, we present some useful properties of Mahgoub and Elzaki transforms like the linearity property, change of scale property, first shifting theorem and convolution theorem.

Linearity property of Mahgoub and Elzaki transforms:

5.3 Linearity property of Mahgoub transforms :

If Mahgoub transform of functions $F_1(t)$ and $F_2(t)$ are $f_1(v)$ and $f_2(v)$ respectively then Mahgoub transform of $[aF_1(t) + bF_2(t)]$ is given by $[af_1(v) + bf_2(v)]$ where a and b are arbitrary constants.

5.4 Linearity property of Elzaki transforms :

If Elzaki transform of functions $F_1(t)$ and $F_2(t)$ are $T_1(v)$ and $T_2(v)$ respectively then Elzaki transform of $[aF_1(t) + bF_2(t)]$ is given by $[aT_1(v) + bT_2(v)]$ where a and b are arbitrary constants.

Change of scale property of Mahgoub and Elzaki transforms:

5.5 Change of scale property of Mahgoub transforms :

If Mahgoub transform of function $F(t)$ is $f(v)$ then Mahgoub transform of function $F(at)$ is

$$f\left(\frac{v}{a}\right)$$

5.6. Change of scale property of Elzaki transforms :

If Elzaki transform of function $F(t)$ is $T(v)$ then Elzaki transform of function $F(at)$ is

$$\frac{1}{a^2} T(av)$$

Shifting property of Mahgoub and Elzaki transforms:

5.7 Shifting property of Mahgoub transforms

If Mahgoub transform of function $F(t)$ is $f(v)$ then Mahgoub transform of function $e^{at}F(t)$ is $\frac{v}{v-a} f(v-a)$

5.8 Shifting property of Elzaki transforms:

If Elzaki transform of function $F(t)$ is $T(v)$ then Mahgoub transform of function $e^{at}F(t)$ is $(1-av)f\left(\frac{v}{1-av}\right)$

Convolution Theorem for Mahgoub and Elzaki Transforms

5.9 Convolution theorem for Mahgoub transforms :

If Mahgoub transform of functions $F_1(t)$ and $F_2(t)$ are $f_1(v)$ and $f_2(v)$ respectively then Mahgoub transform of their convolution $F_1(t)*F_2(t)$ is given by $M\{F_1(t)*F_2(t)\} = \left(\frac{1}{v}\right)M\{F_1(t)\}M\{F_2(t)\} = \left(\frac{1}{v}\right)f_1(v)f_2(v)$ where $F_1(t)*F_2(t) = \int_0^t F_1(t-x)F_2(x)dx = \int_0^t F_1(x)F_2(t-x)dx$ is defined by

$$\int_0^t F_1(t-x)F_2(x)dx = \int_0^t F_1(x)F_2(t-x)dx$$

5.10 Convolution theorem for Elzaki transforms :

If Elzaki transform of functions $F_1(t)$ and $F_2(t)$ are $T_1(v)$ and $T_2(v)$ respectively then Mahgoub transform of their convolution $F_1(t)*F_2(t)$ is given by $E\{F_1(t)*F_2(t)\} = \left(\frac{1}{v}\right)E\{F_1(t)\}E\{F_2(t)\} = \left(\frac{1}{v}\right)T_1(v)T_2(v)$ where $F_1(t)*F_2(t) = \int_0^t F_1(t-x)F_2(x)dx = \int_0^t F_1(x)F_2(t-x)dx$ is defined by

$$\int_0^t F_1(t-x)F_2(x)dx = \int_0^t F_1(x)F_2(t-x)dx$$

VI. MAHGOUB AND ELZAKI TRANSFORMS OF THE DERIVATIVES OF THE FUNCTION

6.1 Mahgoub transforms of the derivatives of the function $F(t)$

If $M\{F(t)\} = f(v)$ then
 $M\{F'(t)\} = vf(s) - vF(0)$
 $M\{F''(t)\} = v^2 f(s) - v^2F(0) - vF'(0)$

In general,
 $M\{F^{(n)}(t)\} = v^n f(v) - v^n F(0) - v^{n-1}F'(0) - \dots - vF^{(n-1)}(0)$

6.2 Elzaki transforms of the derivatives of the function $F(t)$

If $E\{F(t)\} = T(v)$ then
 $E\{F'(t)\} = \frac{1}{v} T(v) - vF(0)$

$$E\{F''(t)\} = \frac{1}{v^2} T(v) - F(0) - vF'(0)$$

In general,

$$E\{F^{(n)}(t)\} = \frac{1}{v^n} T(v) - \frac{1}{v^{n-2}} F(0) - \frac{1}{v^{n-3}} F'(0) - \dots - vF^{(n-1)}(0)$$

VII. MAHGOUB AND ELZAKI TRANSFORMS OF FREQUENTLY USED FUNCTIONS

Table: 1

| S:No | $F(t)$ | $M\{F(t)\}$ | $E\{F(t)\}$ |
|------|----------------|---------------------------|-------------------------|
| 1. | 1 | 1 | v^2 |
| 2. | t | $\frac{1}{v}$ | v^3 |
| 3. | t^2 | $\frac{2!}{v^2}$ | $2! v^4$ |
| 4. | $t^n, n \in N$ | $\frac{n!}{v^n}$ | $n! v^{n+2}$ |
| 5. | $t^n, n > -1$ | $\frac{\Gamma(n+1)}{v^n}$ | $\Gamma(n+1)v^{n+2}$ |
| 6. | e^{at} | $\frac{v}{v-a}$ | $\frac{v^2}{1-av}$ |
| 7. | $\sin at$ | $\frac{av}{v^2+a^2}$ | $\frac{av^3}{1+a^2v^2}$ |
| 8. | $\cos at$ | $\frac{v^2}{v^2+a^2}$ | $\frac{v^2}{1+a^2v^2}$ |
| 9. | $\sin hat$ | $\frac{av}{v^2-a^2}$ | $\frac{av^3}{1-a^2v^2}$ |
| 10. | $\cos hat$ | $\frac{v^2}{v^2-a^2}$ | $\frac{v^2}{1-a^2v^2}$ |

VIII. Inverse Mahgoub and Elzaki Transforms of Frequently Used Functions:

Table:2

| S:No | $f(v)$ | $F(t) = M^{-1}\{f(v)\} = E^{-1}\{T(v)\}$ | $T(v)$ |
|------|-----------------|--|--------|
| 1. | 1 | 1 | v^2 |
| 2. | $\frac{1}{v}$ | t | v^3 |
| 3. | $\frac{1}{v^2}$ | $\frac{t^2}{2}$ | v^4 |
| 4. | | | |

| | | | |
|-----|-----------------------|-----------------------------------|------------------------|
| | $\frac{1}{v^n}$ | $\frac{t^n}{n!}, n \in N$ | v^{n+2} |
| 5. | $\frac{1}{v^n}$ | $\frac{t^n}{\Gamma(n+1)}, n > -1$ | v^{n+2} |
| 6. | $\frac{v}{v-a}$ | e^{at} | $\frac{v^2}{1-av}$ |
| 7. | $\frac{v}{v^2+a^2}$ | $\frac{\sin at}{a}$ | $\frac{v^3}{1+a^2v^2}$ |
| 8. | $\frac{v^2}{v^2+a^2}$ | $\cos at$ | $\frac{v^2}{1+a^2v^2}$ |
| 9. | $\frac{v}{v^2-a^2}$ | $\frac{\sinh at}{a}$ | $\frac{v^3}{1-a^2v^2}$ |
| 10. | $\frac{v^2}{v^2-a^2}$ | $\cosh at$ | $\frac{v^2}{1-a^2v^2}$ |

IX.Applications of Mahgoub and Elzaki Transforms for Solving System of Differential Equations

In this section, some numerical applications are given to solve the systems of differential equations using Mahgoub and Elzaki transforms.

1. Consider a system of linear ordinary differential equations

$$\frac{dx}{dt} + y = 2 \cos t, \quad x + \frac{dy}{dt} = 0 \tag{1}$$

with $x(0) = 0, y(0) = 1$.

Solution Using Mahgoub Transform Method

Taking Mahgoub transform of system of equation(1), we have

$$M\left(\frac{dx}{dt}\right) + M(y) = 2M(\cos t) \tag{2}$$

$$M(x) + M\left(\frac{dy}{dt}\right) = 0 \tag{3}$$

Now by using the property, Mahgoub transform of the derivatives of the function, in equation (2) and (3), we have

$$vM(x) - vx(0) + M(y) = \frac{2v^2}{v^2 + 1} \tag{4}$$

$$M(x) + vM(y) - vy(0) = 0 \tag{5}$$

Apply the initial condition in the above equation (4) and (5)

$$vM(x) + M(y) = \frac{2v^2}{v^2 + 1} \tag{6}$$

$$M(x) + vM(y) = 0 \tag{7}$$

Solving the above system of equation (6) and (7) for $M(x)$ and $M(y)$, we get

$$M(x) = \frac{v}{v^2 + 1} \quad \text{and} \quad M(y) = \frac{v^2}{v^2 + 1} \tag{8}$$

Taking inverse Mahgoub transform on the system of equation (8) , we get
 $x = \sin t$ and $y = \cos t$
 Which is the required solution.

Solution Using Elzaki Transform Method

Taking Elzaki transform of system (1), we have

$$E\left(\frac{dx}{dt}\right) + E(y) = 2E(\cos t) \tag{9}$$

$$E(x) + E\left(\frac{dy}{dt}\right) = 0 \tag{10}$$

Now by using the property, Elzaki transform of the derivatives of the function, in the equation(9) and (10), we have

$$\frac{1}{v} E(x) - vx(0) + E(y) = \frac{2v^2}{1+v^2} \tag{11}$$

$$E(x) + \frac{1}{v} E(y) - vy(0) = 0 \tag{12}$$

Apply the initial condition in the above equation (11) and (12),we get

$$\frac{1}{v} E(x) + E(y) = \frac{2v^2}{1+v^2} \tag{13}$$

$$E(x) + \frac{1}{v} E(y) = 0 \tag{14}$$

Solving the above system of equation (13) and (14) for $E(x)$ and $E(y)$, we get

$$E(x) = \frac{v^3}{1+v^2} \text{ and } E(y) = \frac{v^2}{1+v^2} \tag{15}$$

Taking inverse Elzaki transform on the system of equation (15) , we get
 $x = \sin t$ and $y = \cos t$
 which is the required solution.

2. Consider a system of linear ordinary differential equations

$$\frac{d^2x}{dt^2} + 3x - 2y = 0 \tag{16}$$

and $\frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} - 3x + 5y = 0 \tag{17}$

with $x(0) = 0, y(0) = 0, x'(0) = 3, y'(0) = 2$

Solution Using Mahgoub Transform Method

Taking Mahgoub transform of system of equation (16) and (17), we have

$$M\left(\frac{d^2x}{dt^2}\right) + 3M(x) - 2M(y) = 0 \tag{18}$$

$$M\left(\frac{d^2x}{dt^2}\right) + M\left(\frac{d^2y}{dt^2}\right) - 3M(x) + 5M(y) = 0 \tag{19}$$

Now using the property, Mahgoub transform of the derivatives of the function, in (18)and (19), we have

$$v^2M(x) - v^2x(0) - vx'(0) + 3M(x) - 2M(y) = 0 \tag{20}$$

$$v^2M(x) - v^2x(0) - vx'(0) + v^2M(y) - v^2y(0) - vy'(0) - 3M(x) + 5M(y) = 0 \quad (21)$$

Apply the initial condition in the above equation (20) and (21), we get

$$(v^2 + 3)M(x) - 2M(y) = 3v \quad (22)$$

$$(v^2 - 3)M(x) + (v^2 + 5)M(y) = 5v \quad (23)$$

Solving the above system of equation (22) and (23) for $M(x)$ and $M(y)$, we get

$$M(x) = \frac{11}{4} \left[\frac{v}{v^2 + 1} \right] + \frac{1}{4} \left[\frac{v}{v^2 + 9} \right] \quad (24)$$

$$M(y) = \frac{11}{4} \left[\frac{v}{v^2 + 1} \right] - \frac{3}{4} \left[\frac{v}{v^2 + 9} \right] \quad (25)$$

Taking inverse Mahgoub transform on the system of equation (24) and (25), we get

$$x = \frac{11}{4} \sin t + \frac{1}{12} \sin 3t$$

$$y = \frac{11}{4} \sin t - \frac{1}{4} \sin 3t$$

Solution Using Elzaki Transform Method

Taking Elzaki transform of system (16) and (17), we have

$$E \left(\frac{d^2x}{dt^2} \right) + 3E(x) - 2E(y) = 0 \quad (26)$$

$$E \left(\frac{d^2x}{dt^2} \right) + E \left(\frac{d^2y}{dt^2} \right) - 3E(x) + 5E(y) = 0 \quad (27)$$

Now using the property, Elzaki transform of the derivatives of the function, in (26) and (27), we have

$$\frac{1}{v^2} E(x) - x(0) - vx'(0) + 3E(x) - 2E(y) = 0 \quad (28)$$

$$\frac{1}{v^2} E(x) - x(0) - vx'(0) + \frac{1}{v^2} E(y) - y(0) - vy'(0) - 3E(x) + 5E(y) = 0 \quad (29)$$

Apply the initial condition in the above equation (28) and (29), we have

$$\left(\frac{1}{v^2} + 3 \right) E(x) - 2E(y) = 3v \quad (30)$$

$$\left(\frac{1}{v^2} - 3 \right) E(x) + \left(\frac{1}{v^2} + 5 \right) E(y) = 5v \quad (31)$$

Solving the above system of equation (30) and (31) for $M(x)$ and $M(y)$, we get

$$E(x) = \frac{11}{4} \left[\frac{v^3}{1 + v^2} \right] + \frac{1}{4} \left[\frac{v^3}{1 + 9v^2} \right] \quad (32)$$

$$E(y) = \frac{11}{4} \left[\frac{v^3}{1 + v^2} \right] - \frac{3}{4} \left[\frac{v^3}{1 + 9v^2} \right] \quad (33)$$

Taking inverse Elzaki transform on the system of equation (32) and (33), we get

$$x = \frac{11}{4} \sin t + \frac{1}{12} \sin 3t$$

$$y = \frac{11}{4} \sin t - \frac{1}{4} \sin 3t$$

which is the required solution.

X.Test

Figure 1, 2 and 3 gives the illustrations of the connections between the transforms

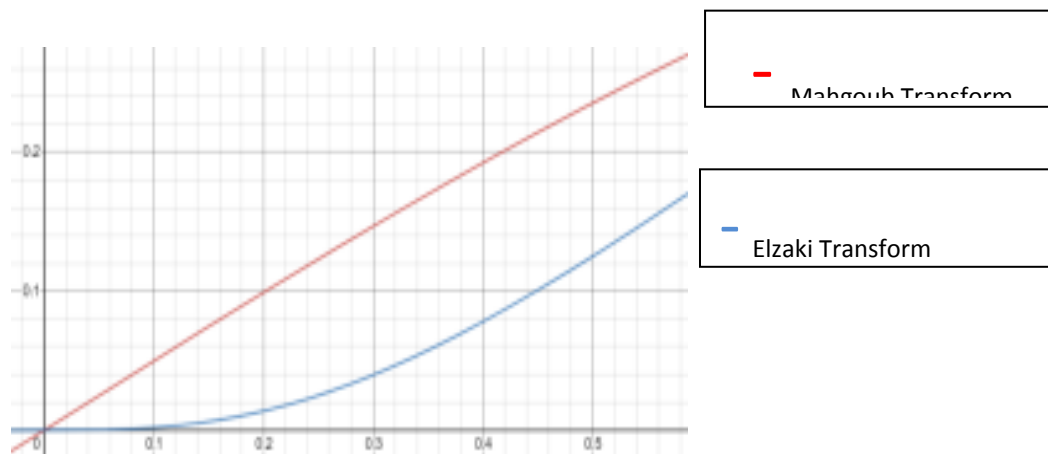


Fig.1 . Plots for Mahgoub and Elzaki transforms of $F(t) = \sin 2t$

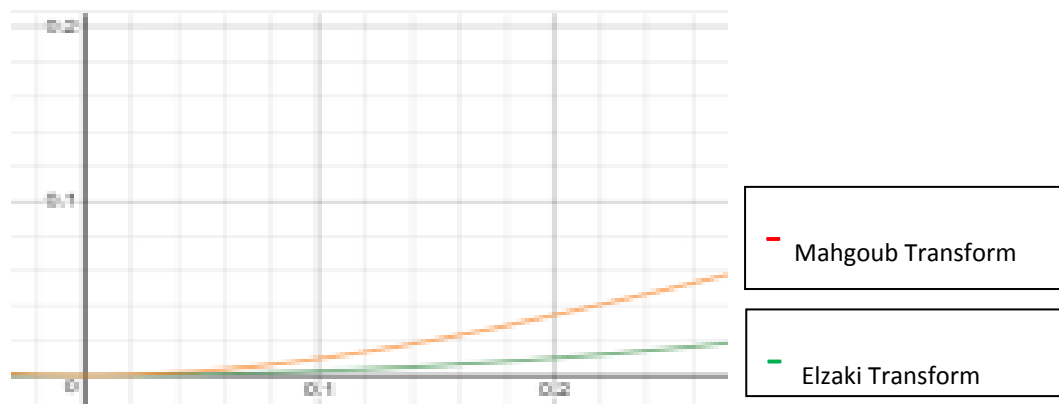


Fig.2 . Plots for Mahgoub and Elzaki transforms of $F(t) = \cos 2t$

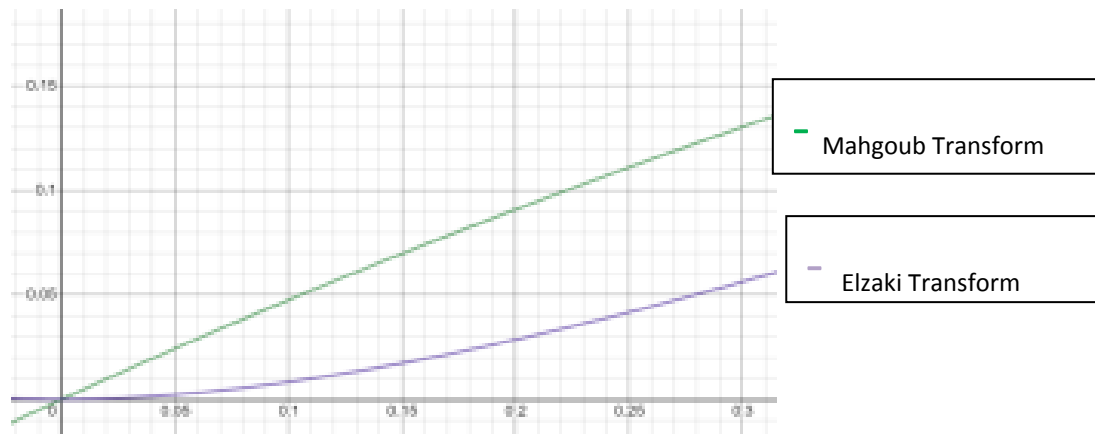


Fig.3 . Plots for Mahgoub and Elzaki transforms of $F(t) = e^{-2t}$

Conclusion:

Most of the challenges in the world do not get solved because people trying to address those problems focus too much on the problem statement. At other times, the problem statement is overlooked and there is too much stress to find a solution.

Design thinking helps to gain a balance between the problem statement and the solution developed. A design-oriented mindset is not problem focused, but solution focused and action oriented. It has to involve both analysis and imagination. Design thinking is the way of resolving issues and dissolving problematic situations by the help of design.

Some test functions are considered as examples and illustrated graphically with the help of Mathematica software. In this paper, we have successfully discussed the comparative study of Mahgoub and Elzaki transforms using design thinking concept. In application section, we solve systems of differential equations comparatively using both the transforms. The given numerical applications in application section show that both the transforms (Mahgoub and Elzaki transforms) are closely connected to each other.

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