An Algorithmic Approach for finding Minimum Spanning Tree in a Bipolar Spherical Fuzzy Graph

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Abstract

Graph theory is used for modelling of real world network systems such as: water, construction, pattern designs, transport, electricity, internet, etc. In this paper, we propose an algorithm for finding minimum spanning tree of an undirected bipolar graph where the edge lengths are represented by bipolar spherical fuzzy number. To construct the minimum spanning tree of undirected bipolar spherical fuzzy graph, a new algorithm and score function based on matrix approach has been introduced. The proposed method compare with some existing method are also discussed.

Keywords: Bipolar Spherical Fuzzy Graph, Minimum Spanning Tree, Score Function

1. Introduction

Spherical fuzzy set is a generalization of picture fuzzy set and Pythagorean fuzzy set. There is a need of spherical fuzzy set to tackle an interesting scenario emerge when picture fuzzy sets and Pythagorean fuzzy sets both failed to handle. We can study the neutral degree in spherical fuzzy set where as in Pythagorean fuzzy sets and picture fuzzy sets it doesn't. In spherical fuzzy set, membership degrees are gratifying the condition $0 \leq P^2(x) + I^2(x) + N^2(x) \leq 1$ [3,18,19].

Bipolar fuzzy sets is an extension of fuzzy sets, in which positive information represents the possible and negative information represents the impossible or surely false [30]. In bipolar fuzzy sets, the elements are irrelevant are indicated by membership degree zero, the elements are satisfy the corresponding property by (0,1] and the elements are satisfy implicit counter property by [-1,0). On the other hand graphical representation is a convenient way of representing the data in which the objects are vertices and their relations are edges. Fuzzy graph models were developed to describe the uncertain elements but their extension fails if the relation between the nodes in the problem is indeterminate [4-13]. The new concept of bipolar neutrosophic cubic graphs and bipolar spherical fuzzy neutrosophic cubic graphs is introduced and discuss some of their algebraic properties and present minimum spanning tree algorithm with numerical example [1,2].

Prim and Kruskal algorithm are the two most common algorithms for finding the minimum spanning tree including in classical graph theory [15,17,23]. A new theory is introduced called single valued neutrosophic graph theory (SVNGT) and their extensions finds its applications in diverse fields[12]. Ye [26-29] presented a method to obtain minimum spanning tree of a graph where nodes (samples) are represented in the form of single valued neutrosophic set and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived.

Kandasamy [16] introduced a Double-Valued Neutrosophic Minimum Spanning Tree (DVN-MST) clustering algorithm to cluster the data represented by double-valued neutrosophic information. Mandal and Basu [20] proposed a solution approach of the optimum spanning tree problems considering the incompleteness, indeterminacy and inconsistency of the information. Mullai [22] developed the minimum spanning tree problem on a graph in which a bipolar neutrosophic number is associated to each edge as its edge length, and illustrated it by a numerical example.

The main objective of this paper is to present a spherical Kruskal algorithm for searching the cost minimum spanning tree of an undirected graph in which a bipolar spherical fuzzy number is associated to each edge as its edge length.

The rest of the manuscript as follows. The idea of intuitionistic fuzzy sets, Pythagorean fuzzy sets, bipolar neutrosophic sets, spherical fuzzy sets, bipolar spherical fuzzy sets and minimum spanning tree are briefly introduced in section 2. In Section 3, an score function of bipolar spherical fuzzy set and algorithm for finding the minimum spanning tree of spherical fuzzy undirected graph in bipolar environment is defined. In Section 4, An numerical example is presented to illustrate the proposed method. A comparative study with existing method is proposed in Section 5. Finally, the conclusion of the paper is presented in Section 6.

2. Preliminaries

In this section, we study some basic definitions required to define an algorithmic approach for finding Minimum Spanning Tree in a Bipolar Spherical Fuzzy Graph.

Definition 2.1. Let *A* be an IFS in the universe of discourse *X*, shown as follows:

$$A = \{< x,\ \mu_A(x), \nu_A(x) >| x \in X\},$$

where $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and non-membership degree of element *x* belonging to the IFS *A*, respectively. Moreover, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitancy degree of element *x* belonging to the IFS *A*.

Definition 2.2. Let *P* be an PFS in the universe of discourse *X*, shown as follows:

$$P = \{< x,\ \mu_A(x), \nu_A(x) >| x \in X\},$$

where $\mu_P(x): X \to [0,1]$ and $\nu_P(x): X \to [0,1]$ satisfy $0 \le (\mu_A(x))^2 + (\nu_A(x))^2 \le 1$ for all $x \in X$, $\mu_p(x)$ and $\nu_P(x)$ denote the membership degree and non-membership degree of element *x* belonging to the PFS *P*, respectively. Moreover, $\pi_P(x) = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ is called the hesitancy degree of element *x* belonging to the PFS *P*. For convenience, we introduce a Pythagorean fuzzy number denoted by $\beta = P(\mu_\beta, \nu_\beta)$, where $\mu_\beta, \nu_\beta \in [0,1]$ and $0 \le (\mu_\beta)^2 + (\nu_\beta)^2 \le 1$.

Definition 2.3. Bipolar Neutrosophic Set

A bipolar neutrosophic set *A* in *X* is defined as an object of the form

$$P = \{< x, T_P^+(x), I_P^+(x),\ F_P^+(x), T_P^-(x), I_P^-(x),\ F_P^-(x) >| x \in X\},$$

where $T_P^+, I_P^+, F_P^+ : X \to [0,1]$, $T_P^-, I_P^-, F_P^- : X \to [0,1]$.

Definition 2.4. Spherical Fuzzy Set

Let *X* be a universe. Then the set

$$P = \{< x, (T_P(x),\ I_P(x),\ F_P(x)) >| x \in X\},$$

is said to be spherical fuzzy set, where $T_P(x): X \to [0,1]$, $I_P(x): X \to [0,1]$ and $F_P(x): X \to [0,1]$ are said to be degree of positive-membership function of *x* in *X*, degree of neutral-membership function of *x* in *X* and degree of negative-membership function of *x* in *X*, respectively. Also T_P, I_P and F_P satisfy the following condition:

$$(\forall x \in X) \quad (0 \le (T_P(x))^2 + (I_P(x))^2 + (F_P(x))^2 \le 1).$$

Definition 2.5 Bipolar Spherical Fuzzy Set

Let X be a non-empty set. A Bipolar Spherical Fuzzy Set (BSFS)

$$A=\{<x,T_A^P,I_A^P,F_A^P,T_A^N,I_A^N,F_A^N>|\, x\in X\}$$

where $T_A^P,I_A^P,F_A^P : X\to[0,1]$, $T_A^N,I_A^N,F_A^N : X\to[-1,0]$ are the mappings such that $0\le((T_A^P)^2+(I_A^P)^2+(F_A^P)^2)\le 1$ and $0\le((T_A^N)^2+(I_A^N)^2+(F_A^N)^2)\le 1$ and T_A^P denote the positive truth membership function, I_A^P denote the positive indeterminacy membership function, F_A^P denote the positive falsity membership function, T_A^N denote the negative truth membership function, I_A^N denote the negative indeterminacy membership function, F_A^N denote the negative falsity membership function.

Definition 2.6 Minimum Spanning Tree

A minimum spanning tree of a graph G is a spanning tree whose weight is minimum among all spanning trees of the graph G.

3. Bipolar Single-Valued Spherical Fuzzy Graph

In this section, we define score function of bipolar spherical fuzzy set and present a minimum spanning tree problem and discuss it on a graph.

Definition 3.1

Let A be a bipolar spherical fuzzy set, we define a new score function as follows:

$$S(A)=\frac{1}{6}\left\{T^P+1-I^P+1-F^P+1+T^N-I^N-F^N\right\}$$

In the following, we propose Bipolar Spherical Fuzzy Minimum Spanning Tree algorithm [BSFMST]

Step (1): Input bipolar spherical fuzzy adjacency matrix A.

Step (2): Interpret the bipolar spherical fuzzy matrix into score matrix S_{ij} by using score.

Step (3): Redo Step (4) & Step (5) until all (n-1) entries of the matrix of *S(A)* are either marked to zero or all the non-zero entries are marked.

Step (4): Find the score matrix *S(A)* either row-wise or column-wise to find the cost of the corresponding edge e_{ij} in *S(A)* that is the minimum entries in S_{ij}.

Step (5): Set $S_{ij}=0$ if the edge e_{ij} of selected S_{ij} construct a cycle with the previous marked elements of the score matrix *S(A)* else mark S_{ij}.

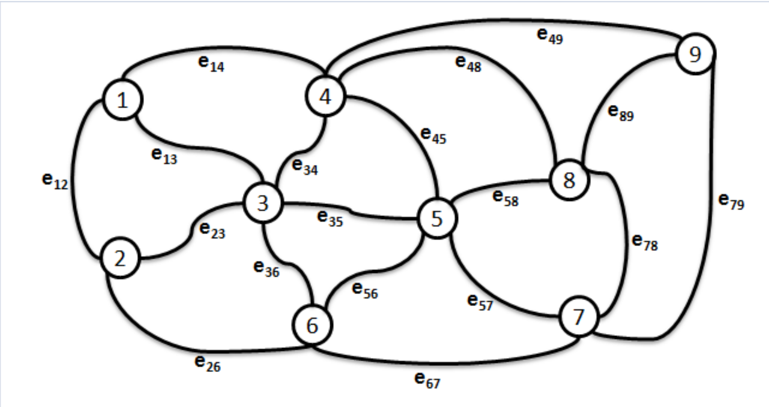
Step (6): Compute minimum cost spanning tree of the graph G by construct the tree T including only the marked elements from the score matrix *S(A)*.

Step (7): End

4. Numerical Example

Assume the graph *G=(V,E)* where *V* be the vertices and *E* be the edge of the graph. Here we have 5 vertices and 7 edges. Erection of the minimum cost spanning tree are discussed as follows

Figure (1): Undirected Graph *G=(V,E)* with 9 vertices and 18 edges



E	Edge length
e_{12}	$\{0.4,0.8,0.3,-0.2,-0.4,-0.5\}$
e_{13}	$\{0.9,0.7,0.8,-0.4,-0.5,-0.8\}$
e_{14}	$\{0.3,0.5,0.5,-0.7,-0.5,-0.2\}$
e_{23}	$\{0.9,0.1,0.5,-0.8,-0.6,-0.3\}$
e_{26}	$\{0.6,0.7,0.5,-0.5,-0.4,-0.3\}$
e_{34}	$\{0.3,0.4,0.2,-0.7,-0.6,-0.3\}$
e_{35}	$\{0.8,0.9,0.6,-0.6,-0.4,-0.3\}$
e_{36}	$\{0.4,0.5,0.1,-0.3,-0.2,-0.4\}$
e_{45}	$\{0.3,0.4,0.7,-0.6,-0.3,-0.6\}$
e_{48}	$\{0.6,0.5,0.3,-0.5,-0.4,-0.8\}$
e_{49}	$\{0.3,0.4,0.2,-0.5,-0.6,-0.8\}$
e_{56}	$\{0.8,0.5,0.2,-0.5,-0.3,-0.8\}$

e_{57}	$\{0.1,0.1,0.8,-0.3,-0.4,-0.5\}$
e_{58}	$\{0.3,0.4,0.6,-0.4,-0.7,-0.6\}$
e_{67}	$\{0.5,0.4,0.3,-0.4,-0.5,-0.9\}$
e_{78}	$\{0.6,0.7,0.8,-0.6,-0.4,-0.2\}$
e_{79}	$\{0.4,0.1,0.3,-0.2,-0.5,-0.7\}$
e_{89}	$\{0.1,0.9,0.5,-0.5,-0.2,-0.8\}$

The bipolar spherical fuzzy adjacency matrix A is given below

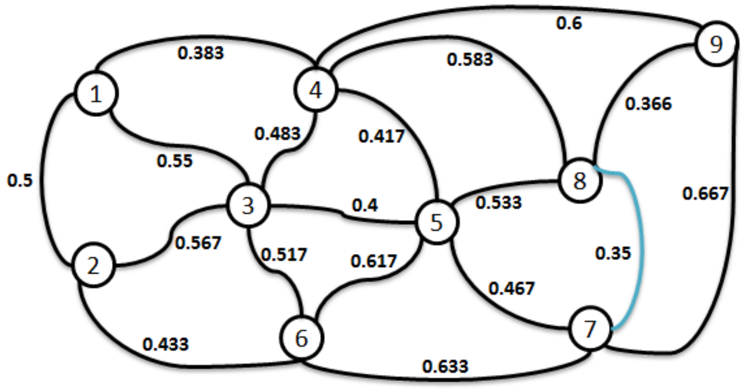
$$S(A) = \begin{pmatrix} 0 & e_{12} & e_{13} & e_{14} & 0 & 0 & 0 & 0 & 0 \\ e_{12} & 0 & e_{23} & 0 & 0 & e_{26} & 0 & 0 & 0 \\ e_{13} & e_{23} & 0 & e_{34} & e_{35} & e_{36} & 0 & 0 & 0 \\ e_{14} & 0 & e_{34} & 0 & e_{45} & 0 & 0 & e_{48} & e_{49} \\ 0 & 0 & e_{35} & e_{45} & 0 & e_{56} & e_{57} & e_{58} & 0 \\ 0 & e_{26} & e_{36} & 0 & e_{56} & 0 & e_{67} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{57} & e_{67} & 0 & e_{78} & e_{79} \\ 0 & 0 & 0 & e_{48} & e_{58} & 0 & e_{78} & 0 & e_{89} \\ 0 & 0 & 0 & e_{49} & 0 & 0 & e_{79} & e_{89} & 0 \end{pmatrix}$$

Thus, the score matrix using the score function

Figure (2): Score Matrix

$$\begin{pmatrix} 0 & 0.5 & 0.55 & 0.383 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.567 & 0 & 0 & 0.433 & 0 & 0 & 0 \\ 0.55 & 0.567 & 0 & 0.483 & 0.4 & 0.517 & 0 & 0 & 0 \\ 0.383 & 0 & 0.483 & 0 & 0.417 & 0 & 0 & 0.583 & 0.6 \\ 0 & 0 & 0.4 & 0.417 & 0 & 0.617 & 0.467 & 0.533 & 0 \\ 0 & 0.433 & 0.517 & 0 & 0.617 & 0 & 0.633 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.467 & 0.633 & 0 & 0.35 & 0.667 \\ 0 & 0 & 0 & 0.583 & 0.533 & 0 & 0.35 & 0 & 0.366 \\ 0 & 0 & 0 & 0.6 & 0 & 0 & 0.667 & 0.366 & 0 \end{pmatrix}$$

Figure (3): The selected edge (7,8) in G

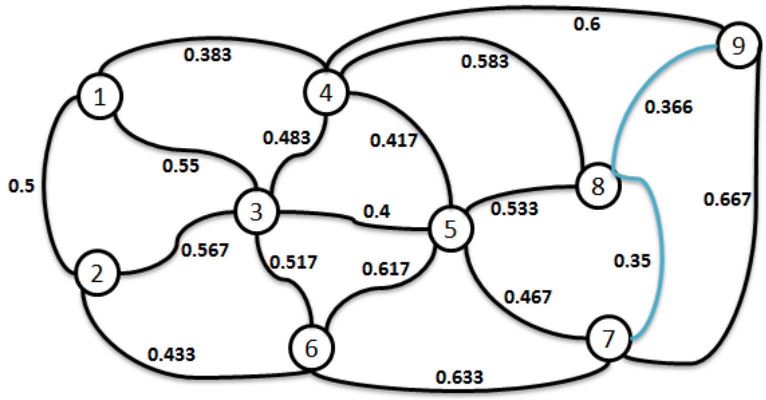


It is clearly observed that 0.35 is selected in Figure (2) is the least value and colored corresponding edge (7,8) is given in Figure (3).

Figure (4): Score Matrix

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (5): The selected edge (8,9) in G

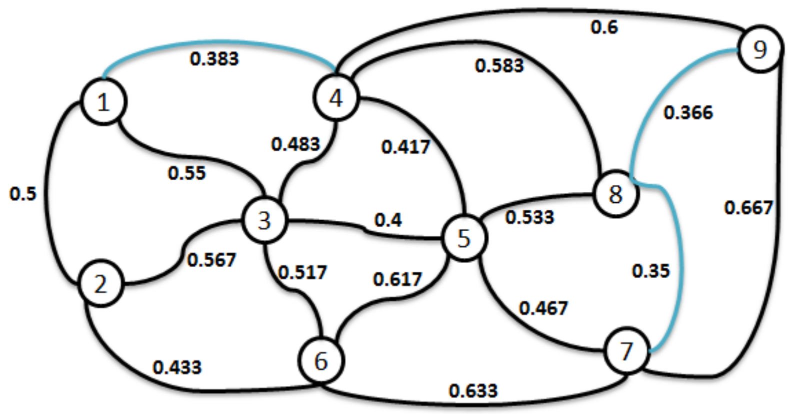


According to the Figure (4) & Figure (5), the next non-zero minimum entry is 0.366 is selected and the corresponding edge (8,9) is highlighted.

Figure (6): Score Matrix

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (7): The selected edge (1,4) in G

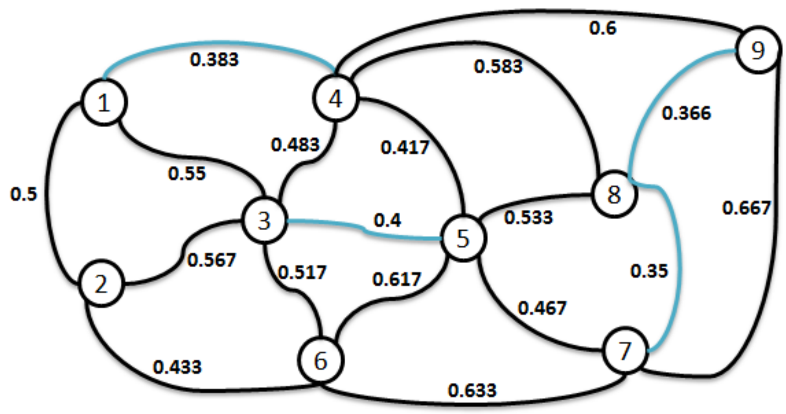


According to the Figure (6) & Figure (7), the next non-zero minimum entry is 0.383 is marked and the corresponding edge (1,4) is highlighted.

Figure (8): Score Matrix

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (9): The selected edge (3,5) in G

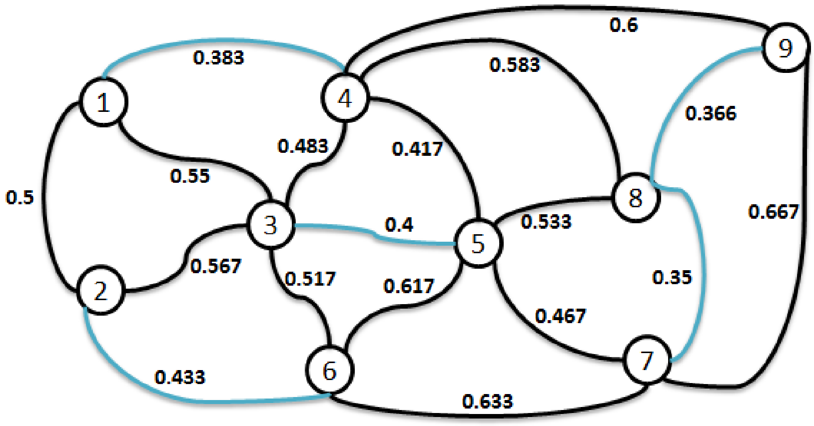


It is clearly observed that from Figure (8), the next non-zero minimum entry is 0.4 is selected and the corresponding edge (3,5) is highlighted in Figure (9).

Figure (10): Score Matrix

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (11): The selected edge (2,6) in G

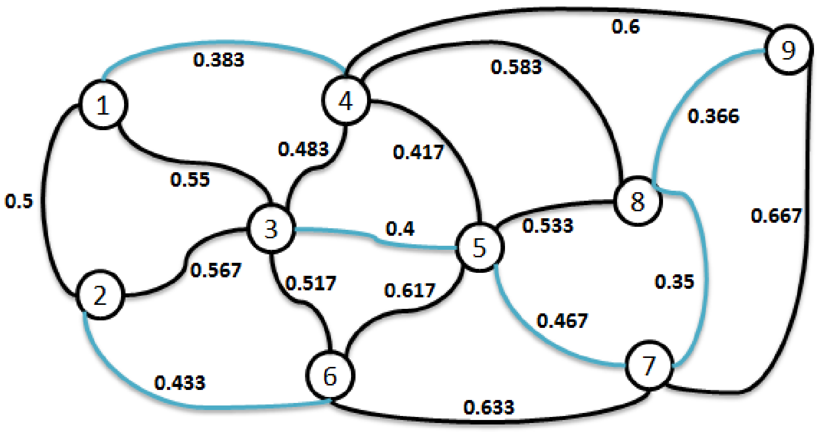


It is clearly observed that from Figure (10), the next non-zero minimum entry is 0.417 is selected but while drawing edge it produces the cycle. So we reject and select the next least value 0.433 and colored corresponding edge (2,6) in Figure (11).

Figure (12): Score Matrix

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (13): The selected edge (5,7) in G

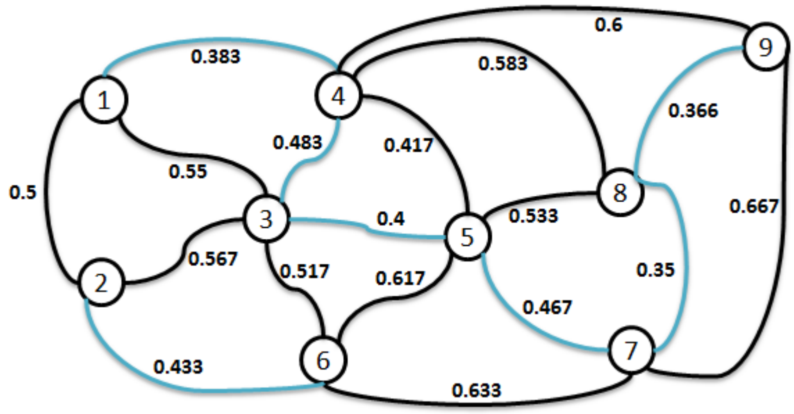


It is clearly observed that from Figure (12), the next non-zero minimum entry is 0.467 is selected and the corresponding edge (5,7) is highlighted in Figure (13).

Figure (14): Score Matrix

0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (15): The selected edge (3,4) in G

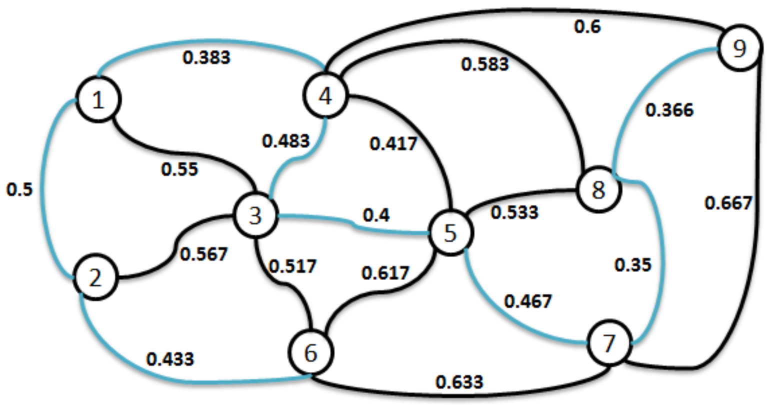


It is clearly observed that from Figure (14), the next non-zero minimum entry is 0.483 is selected and the corresponding edge (3,4) is highlighted in Figure (15).

Figure (16): Score Matrix

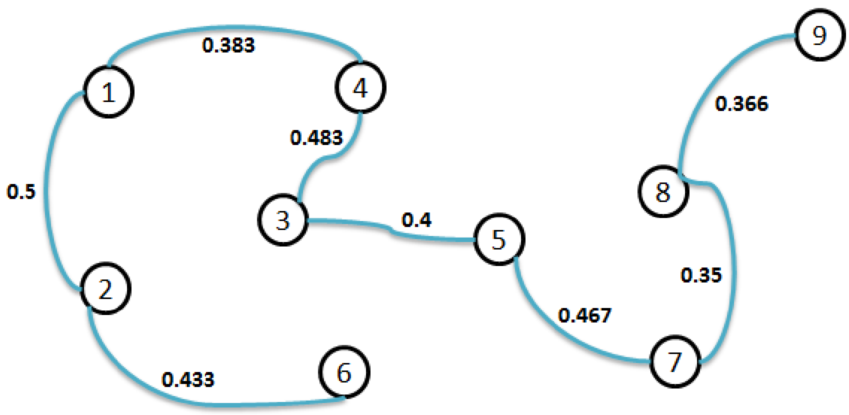
0	0.5	0.55	0.383	0	0	0	0	0
0.5	0	0.567	0	0	0.433	0	0	0
0.55	0.567	0	0.483	0.4	0.517	0	0	0
0.383	0	0.483	0	0.417	0	0	0.583	0.6
0	0	0.4	0.417	0	0.617	0.467	0.533	0
0	0.433	0.517	0	0.617	0	0.633	0	0
0	0	0	0	0.467	0.633	0	0.35	0.667
0	0	0	0.583	0.533	0	0.35	0	0.366
0	0	0	0.6	0	0	0.667	0.366	0

Figure (17): The selected edge (1,2) in G



It is clearly observed that from Figure (16), the next non-zero minimum entry is 0.5 is selected and colored corresponding edge (1,2) in Figure (17).

Figure (18): The final path of minimum spanning tree is represented



Using the above steps, the crisp minimum cost spanning tree is 3.382 and the final path of minimum spanning tree is $\{6,2\},\{2,1\},\{1,4\},\{4,3\},\{3,5\},\{5,7\},\{7,8\},\{8,9\}$.

5.Comparative Study

In this section, the same process is carried out by the existing algorithm of Mullai et al [22]. The results obtained in different iterations are illustrated below:

Iteration 1: $C_1 = \{9\}$ and $\overline{C_1} = \{1,2,3,4,5,6,7,8\}$

Iteration 2: $C_1 = \{9,8\}$ and $\overline{C_1} = \{1,2,3,4,5,6,7\}$

Iteration 3: $C_1 = \{9,8,7\}$ and $\overline{C_1} = \{1,2,3,4,5,6\}$

Iteration 4: $C_1 = \{9,8,7,5\}$ and $\overline{C_1} = \{1,2,3,4,6\}$

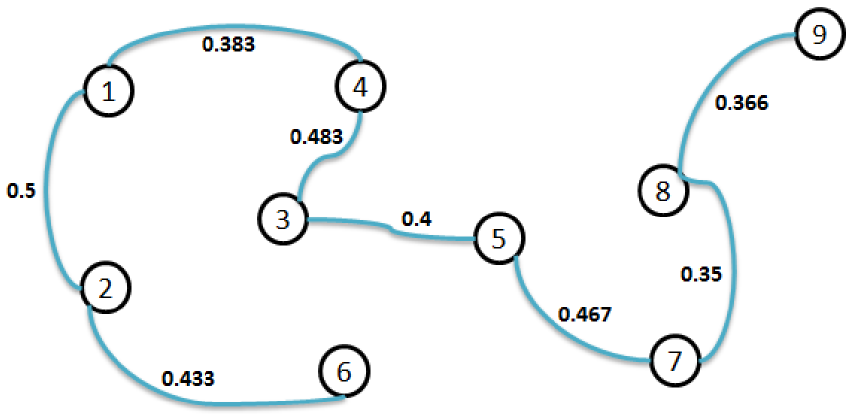
Iteration 5: $C_1 = \{9,8,7,5,3\}$ and $\overline{C_1} = \{1,2,4,6\}$

Iteration 6: $C_1 = \{9,8,7,5,3,4\}$ and $\overline{C_1} = \{1,2,6\}$

Iteration 7: $C_1 = \{9,8,7,5,3,4,1\}$ and $\overline{C_1} = \{2,6\}$

Iteration 8: $C_1 = \{9,8,7,5,3,4,1,2\}$ and $\overline{C_1} = \{6\}$

Figure (19): Minimum Spanning Tree obtained by Mullai's algorithm



Finally, we have the single-valued neutrosophic minimum spanning tree $\{9,8\}$, $\{8,7\}$, $\{7,5\}$, $\{5,3\}$, $\{3,4\}$, $\{4,1\}$. $\{1,2\}$, $\{2,6\}$ obtained by existing algorithm is the same as the proposed algorithm.

The advantage of proposed algorithm over Mullai's algorithm is that the new algorithm is matrix based and can be easily computed in MATLAB & R language whereas the existing algorithm is based on edge comparison leads to high computation.

6. Conclusion

Minimum spanning tree have direct applications in the design of circuit, networks, cluster analysis, computer science, field of medical science. This paper considers a minimum spanning tree problem under the environment where the weights of edges are represented by bipolar spherical fuzzy set. The proposed algorithm for minimum spanning tree is better and efficient. The same results have been obtained for minimum spanning tree by using algorithm of Mullai, but the procedure is much simpler in the proposed method. This work can be extended to the case of directed graphs such as bipolar neutrosophic graphs.

Conflicts of Interest: Authors declare that they have no conflict of interest.

7. References

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