Domination and s-Path Domination in Brick product graphs of some odd cycle graphs

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Abstract

A **dominating set** or **dset** in a connected graph G is called a $s-path$ dset, de noted by D_{p_s} if any path of length $s \in G \geq$ one vertex from this dset, where $2 \leq s \leq diamG$. The $s-path$ dominaton number of G denoted by $\gamma_{p_s}(G)$ is the minimal cardinality taken over all $s-path$ dsets of G. In this paper, we determine $\gamma(G)$ and $\gamma_{p_s}(G)$ for some brick product graphs associated with odd cycles.

Key Words: dset, domination number, $s-path$ domination number.
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1 Introduction

By a graph G, we mean a simple, finite and n't directed graph loopless and multiple lines. If $D \subseteq V$ is a *dominating set* of G, if any vertex $\notin D$ is adjoining to few vertex $\in D$. The *domination number* of G de noted by $\gamma(G)$ is the minimal cardinality taken over all dsets of G.

Definition 1.1. *A dominating of G is called a $s-$ path dset, de noted by D_{p_s}, if any path of length $s \in G \geq$ one vertex in this dset, where $2 \leq s \leq diamG$. The $s-$ path dominaton number of G, de noted by $\gamma_{p_s}(G)$.*

From definition 1.1, it is clear that every $s-path$ dset is a dset but the converse is n't true. Also, $|D| \leq |D_{p_s}|$ and hence, $\gamma(G) \leq \gamma_{p_s}(G)$.

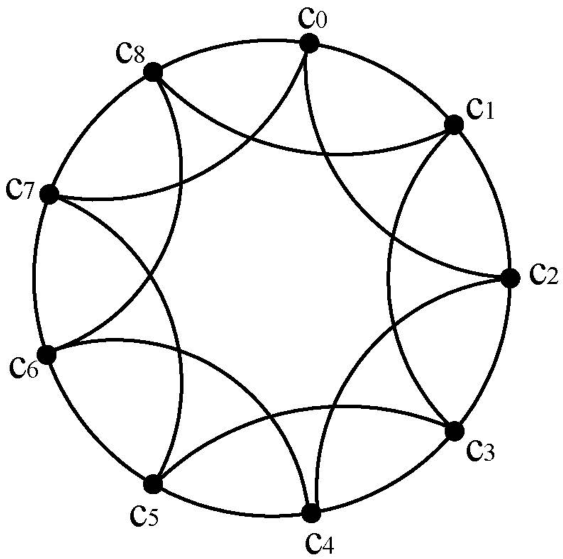

Figure 1: G with $\gamma_{P_2}(G) = 4$

For the graph G in figure 1, the sets $D = \{c_0, c_4\}$, $\{c_1, c_5\}$, $\{c_2, c_7\}$, $\{c_3, c_7\}$ etc. are dsets. Without loss generality if we consider the set $\{c_0, c_4\}$ as the dset, this set can n't be a 2-path dset of G since each of the paths $c_3-c_5-c_6, c_3-c_5-c_7$, $c_3-c_1-c_8$ and $c_1-c_8-c_6$ of length two do n't contain either c_0 or c_4. But, the set $\{c_0, c_3, c_4, c_8\}$ is a 2-path dset and hence $\gamma_{P_2}(G) = 4$. This shows that $|D| < |D_{p_s}|$.

Definition 1.2. *Let k,l and m be positive integers. Let $C_{2l+1} = c_0, c_1, c_2, \cdots, c_{2n}, c_0$ de n'te a cycle of order $2l+1$ $(l > 1)$. The (k,m)-brick product of C_{2l+1}, de n'ted by $C(2l+1, k, m)$ is defined as follows:*

For $k = 1$, we noted that $1 < m < 2l$. Then, $C(2l+1, k, m)$ is acquire from C_{2l+1} by adding chords (c_n, c_{n+r}), $0 \leq n \leq 2l$ where the computation is followed under modulo $2l+1$.

For $k > 1$, $C(2l+1, k, m)$ is acquire by first taking the disjoint union of k copies C_{2l+1} namely $C_{2l+1}(1), C_{2l+1}(2), C_{2l+1}(3), \cdots, C_{2l+1}(k)$ where for each $i = 1, 2, 3, \cdots, k$, $C_{2l+1}(i) = c_{i1}, c_{i2}, c_{i3}, \cdots, c_{i(2l)}, c_{i0}, c_{i1}$.

Further,

Case(i): *If k is odd and $1 < m < 2l$ where m is defined as $m = [(2l+1)n] + 2$, $n \geq 0$, an edge is drawn to join c_{in} to $c_{(i+1)n}$ for both odd or both even $1 \leq i \leq (k-1)$, $0 \leq n \leq 2l$ whereas for each odd $1 \leq i \leq (k-1)$and even $1 \leq n < 2l$ an edge is drawn to join c_{in} to $c_{k(n+1)}$. Finally an edge is drawn to join $c_{i(2l)}$ to $v_{k(2l+m)}$.*

Case(ii): *If k is even and $1 < m < 2l$ where m is defined as $m = [(2l+1)n] + 3$, $n \geq 0$, an edge is drawn to join c_{in} to $c_{(i+1)n}$ for both odd or both even $1 \leq i \leq (k-1)$, $0 \leq n \leq 2l$ whereas for each odd $1 \leq i \leq (k-1)$ and even $1 \leq k < 2l$ an edge is drawn to join c_{in} to $c_{k(n+2)}$. Finally an edge is drawn to join $c_{i(2l)}$ to $c_{k(2l+m)}$.*

The graph G illustrated in figure 1.1 is the brick product graph $C(9,1,2)$.

2 Results

A minimal dset satisfies the following result which can be found in [7].

Theorem 2.1. *A dset D is a minimal dset $\Leftrightarrow$ for every vertex $a \in D$, one of the following conditions hold.*

i) $degree(a) = 0$, $a \in D$

ii) $\exists$ a vertex b in $V - D$ such that $N(b) \cap D$ is equal to $\{a\}$.

For a minimal $s - path$ dset, we have the following result.

Theorem 2.2. *A dset D_{p_s} is a minimal $s - path$ dset $(s \geq 2)$ $\Leftrightarrow$ if for each vertex u in D_{p_s}, one of the following conditions hold.*

1. *$degree(u) = 0$, $u \in D_{p_s}$*
2. *$\exists$ a vertex $v \in V - D_{p_s}$ such that $N(v) \cap D_{p_s}$ is equal to $\{u\}$*
3. *If G is k- connected, $k > 1$ and $v_i, v_j \in D_{p_s}$, then $< D_{p_s} >$ is a disconnected graph and each vertex of $D_{p_s} - \{u\}$ belongs to some cycle in G.*

Proof. Let D_{p_s} be a minimal $s - path$ dset of G. Then for every vertex $u \in D_{p_s}$ if the set $D_{p_s} - \{u\}$ is n't an s-path dset in G, it follows that either $degree(u) = 0$ of D_{p_s} or $\exists$ a vertex $v \in V - D_{p_s}$ such that$N(v) \cap D_{p_s} = \{u\}$. If G is k- connected and $k > 1$, then for $s \geq 2$, every vertex of $D_{p_s} - \{u\}$ belongs to some cycle in G and we have the following two possible cases.

Case 1: Let $v_i, v_j \in D_{p_s}$ such that $d(v_i, v_j) = 1$. Then, $< D_{p_s} >$ is a disconnected graph with one component as K_2 and the remaining components are isolated vertices.

Case 2: Let $v_i, v_j \in D_{p_s}$ such that $d(v_i, v_j) \geq 1$. Then $< D_{p_s} >$ is a disconnected graph in which all components are isolated vertices.

Conversely, let D_{p_s} be an $s - path$ dset satisfying the conditions above. For the purpose of a contradiction, let us assume that D_{p_s} is n't minimal. Then there must exist a vertex $u \in D_{p_s}$ such that $D_{p_s} - \{u\}$ is also an s- path dset. Hence, for atleast one vertex $v \in D_{p_s} - \{u\}$, there must be a path connecting u and v in G, so that $\{u\}$ can n't be an isolated vertex of D_{p_s} and hence condition 1 fails. Also, every vertex in $V - D_{p_s}$ lies in some path connecting atleast one vertex in $D_{p_s} - \{u\}$ so that conditions 2 also fails. For condition 3, it is easy to observe that $\{u\}$ lies in some cycle of G along with the vertices of $V - (D_{p_s} - \{u\})$. So condition 3 also fails. This contradicts the fact the $D_{p_s} - \{u\}$ also a minimal s-path dset.

Hence the proof. □

Theorem 2.3. *If G is a graph with no isolated vertices, then $\gamma_{p_s}(G) \leq \frac{n}{2}$ (where n is the number of vertices in G).*

Proof. Let D_{p_s} be a minimal s-path dset of G. Then, every vertex in D_{p_s} is adjoining to at least one vertex in $V - D_{p_s}$. Hence, $V - D_{p_s}$ is a dset and $\gamma(G) \leq \gamma_{p_s}(G) \leq min\{|D_{p_s}|, |V - D_{p_s}|\} \leq \frac{n}{2}$.

Hence the proof. □

In the next result, we provide the upper bound for the $s - path$ domination number. The proof of this theorem is an immediate consequence.

Theorem 2.4. *For any graph G, $\gamma_{p_s}(G) \leq \lceil \frac{n+1-(\delta(G)-1)\frac{\Delta(G)}{\delta(G)}}{2} \rceil$.*

Since each vertex in G dominates atmost itself and $\Delta(G)$, we have the following result.

Theorem 2.5. *For any graph G, $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma_{p_s}(G)$.*

For the graph $C(2n + 1, 1, 2)$, we have the following theorem.

Theorem 2.6. *For $l \geq 3$, $\gamma(C(2l + 1, 1, 2)) = \lceil \frac{2l+1}{5} \rceil$.*

Proof. Let $G = C(2l + 1, 1, 2)$. We consider the vertex set as $V(G) = \{v_0, v_1, \cdots, v_{2l}\}$ and the edge set as $E(G) = \{e_i : 1 \leq i \leq 2l + 1\} \cup \{e_i^{'} : 1 \leq i \leq 2l + 1\}$, where e_i is the *cycle edge* (v_{i-1}, v_i) and $e_i^{'}$ is the *brick edge* (v_n, v_{n+m}), $n = 0, 1, \cdots, l$. Here $n + m$ is computed modulo $2l + 1$.

For $l = 3, 4, 5, 6, 7, 8, 9$ and 10,

the sets $D = \{v_0, v_4\}, \{v_0, v_5, v_8\}, \{v_0, v_5, v_{10}\}, \{v_0, v_5, v_{10}, v_{14}\}, \{v_0, v_5, v_{10}, v_{15}\}, \{v_0, v_5, v_{10}, v_{15}, v_{18}\}$ are the minimal dsets and hence $\gamma(G) = 2, 3, 3, 4, 4$ and 5 respectively.

Let $l \geq 11$.

Consider the set $D = D_1 \cup D_2$, where $D_1 = \{v_{5j}\}$, $0 \leq j \leq \lceil \frac{l}{3} \rceil$

and

$$D_2 = \begin{cases} v_{2l-2}, & l \equiv 0, 1, 3(mod5) \\ v_{2l-3}, & l \equiv 2(mod5) \\ v_{2l-4}, & l \equiv 4(mod5) \end{cases}$$

This dset D is a minimal, for each vertex $a \in D$, $D - \{a\}$ is n't a dset. Hence, few vertex $b \in V - D \cup \{a\}$ is n't dominated by any vertex in $D \cup \{a\}$. If b is non adjoining to vertex $a \in D$, then b is dominated by $D - \{a\}$. which is a contradiction, Hence the vertex b is adjoining only to vertex $a \in D$.

Therefore, $|D| = \lceil \frac{2l+1}{5} \rceil$, it follows that $\gamma(G) = \lceil \frac{2l+1}{5} \rceil$.

Hence the proof.

□

Theorem 2.7. *For* $l \geq 3$, $\gamma(C(2l+1, 1, 3)) = \begin{cases} \lceil \frac{2l+1}{5} \rceil + 1, & 2l+1 \equiv 4(mod5) \\ \lceil \frac{2l+1}{5} \rceil, & otherwise \end{cases}$.

Proof. Let $G = C(2l+1, 1, 3)$, $V(G)$ and $E(G)$ are as theorem 2.6.

For $l = 3$, the set $D = \{v_0, v_2\}$ is a minimal dset and hence $\gamma(G)$= 2 and for $l = 4, 5$ and 6 the dset $D = \{v_0, v_5, v_8\}$ is a minimal and hence $\gamma(G)$= 3. For $l = 7$ and 8 the sets $D = \{v_0, v_5, v_{10}\}$ and $\{v_0, v_5, v_{10}, v_{15}\}$ are the respective minimal dsets and hence $\gamma(G)$= 3.

Let $l \geq 9$.

Case 1: $l = 5j + 4, 5j + 5$, where j= $1, 2, 3, ...$

In this case, we consider the set $D = \{v_{5j}\} \cup \{v_{2l-1}\}$ where $0 \leq j \leq \lceil \frac{2l}{5} \rceil - 1$

Case 2: $l = 5p + 6, 5p + 7, 5p + 8$, where p= $1, 2, 3, ...$

In this case, we consider the set $D = \{v_{5j}\}$ where $0 \leq j \leq \lceil \frac{2l}{5} \rceil$

The dset D in both the cases above is a minimal, for each vertex $a \in D$, $D - \{a\}$ is n't a dset. Hence, few vertex $b \in V - D \cup \{a\}$ is n't dominated by any vertex in $D \cup \{a\}$. If b is non adjoining to vertex $a \in D$, then b is dominated by $D - \{a\}$. which is a contradiction, Hence the vertex b is adjoining only to vertex $a \in D$.

Therefore,

$$|D| = \begin{cases} \lceil \frac{2l+1}{5} \rceil + 1, & 2l+1 \equiv 4(mod5) \\ \lceil \frac{2l+1}{5} \rceil, & otherwise \end{cases},$$

it follows that

$$\gamma(G) = \begin{cases} \lceil \frac{2l+1}{5} \rceil + 1, & 2l+1 \equiv 4(mod5) \\ \lceil \frac{2l+1}{5} \rceil, & otherwise \end{cases}$$

Hence the proof.

□

Theorem 2.8. *For $l \geq 3$, $\gamma(C(2l+1,1,4)) = \begin{cases} 2\lceil \frac{l-2}{3} \rceil, & l = 3j+4, \quad j = 1,2,3,... \\ 2\lfloor \frac{l-2}{3} \rfloor + 1, & otherwise \end{cases}$.*

Proof. Let $G = C(2l+1,1,4)$, $V(G)$ and $E(G)$ are as theorem 2.6.

For $l = 3,4,5,6,7,8,9$ and 10, the sets $D = \{v_0, v_5\}$, $\{v_0, v_7\}$, $\{v_0, v_7, v_9\}$, $\{v_0, v_6, v_7\}$, $\{v_0, v_7, v_9, v_{13}\}$, $\{v_0, v_7, v_9, v_{13}, v_{15}\}$, $\{v_0, v_7, v_9, v_{13}, v_{17}\}$, $\{v_0, v_7, v_9, v_{13}, v_{17}, v_{19}\}$ are the minimal dsets and hence $\gamma(G)$= $2,3,3,3,4,5,5$ and 6 respectively.

Let $l \geq 11$.

Consider the dset $D = \{v_0, v_7, v_9\} \cup \{v_{6j+7}\} \cup \{v_{6p+11}\}$, where $1 \leq j \leq \lceil \frac{l-5}{3} \rceil$, where $1 \leq p \leq \lceil \frac{l-6}{3} \rceil$

This dset D is a minimal, for each vertex $a \in D$, $D - \{a\}$ is n't a dset. Hence, few vertex $b \in V - D \cup \{a\}$ is n't dominated by any vertex in $D \cup \{a\}$. If b is non adjoining to vertex $a \in D$, then b is dominated by $D - \{a\}$. which is a contradiction, Hence the vertex b is adjoining only to vertex $a \in D$.

Therefore,

$$|D| = \begin{cases} 2\lceil \frac{l-2}{3} \rceil, & l = 3j+4, \quad j = 1,2,3,... \\ 2\lfloor \frac{l-2}{3} \rfloor + 1, & otherwise \end{cases},$$

it follows that

$$\gamma(G) = \begin{cases} 2\lceil \frac{l-2}{3} \rceil, & l = 3j+4, \quad j = 1,2,3,... \\ 2\lfloor \frac{l-2}{3} \rfloor + 1, & otherwise \end{cases}.$$

Hence the proof.

□

Theorem 2.9. *For $l \geq 4$, $\gamma_{p_2}(C(2l+1,1,2)) = 2\lceil \frac{l}{2} \rceil$.*

Proof. Let $G = C(2l+1,1,2)$, $V(G)$ and $E(G)$ are as theorem 2.6. For $l = 4$, the dset $D_{p_2} = \{v_0, v_3, v_4, v_8\}$ is a minimal 2-path dset, $\gamma_{p_2}(G)$= 4.

Let $l \geq 5$.

Consider the dset $D_{p_2} = \{v_0\} \cup \{v_{4j-1}\} \cup \{v_{4p}\} \cup v_{2l}$, where $1 \leq j, p \leq \lceil \frac{l}{2} \rceil - 1$. This dset is a minimal 2-path dset since every vertex of $D_{p_2} - \{u\}$ belongs to some cycle in G. Let $v_i, v_j \in D_{p_2}$ such that $d(v_i, v_j)= 1$, then, $< D_{p_2} >$ is a disconnected graph with one component as K_2 and the remaining components are isolated vertices. Therefore, $|D_{p_2}| = 2\lceil \frac{l}{2} \rceil$, it follows that $\gamma_{p_2}(G) = 2\lceil \frac{l}{2} \rceil$.

Hence the proof.

□

Theorem 2.10. *For $l \geq 3$, $\gamma_{p_2}(C(2l+1, 1, 3)) = l + 1$.*

Proof. Let $G = C(2l+1, 1, 3)$, $V(G)$ and $E(G)$ are as theorem 2.6. Consider the dset $D_{p_2} = \{v_0, v_2, v_4\} \cup \{v_{2j+3}\}$, where $1 \leq j \leq n-2$. This dset is a minimal 2-path dset since every vertex of $D_{p_2} - \{u\}$ belongs to some cycle in G. Let $v_i, v_j \in D_{p_2}$ such that $d(v_i, v_j)= 1$, then $< D_{p_2} >$ is a disconnected graph with one component as K_2 and the remaining components are isolated vertices. Therefore, $|D_{p_2}| = l + 1$, it follows that $\gamma_{p_2}(G) = l + 1$.

Hence the proof. □

Theorem 2.11. *For $l \geq 3$, $\gamma_{p_2}((C(2l+1, 1, 4)) = l + 1$.*

Proof. Let $G = C(2l+1, 1, 4)$, $V(G)$ and $E(G)$ are as theorem 2.6. Consider the dset $D_{p_2} = \{v_0\} \cup \{v_{2j}\}$, where $1 \leq j \leq l$. This dset D_{p_2} is a minimal 2-path dset since every vertex of $D_{p_2} - \{u\}$ belongs to some cycle in G. Let $v_i, v_j \in D_{p_2}$ such that $d(v_i, v_j)= 1$, then $< D_{p_2} >$ is a disconnected graph with one component as K_2 and the remaining components are isolated vertices. Therefore, $|D_{p_2}| = l + 1$, it follows that $\gamma_{p_2}(G) = l + 1$.

Hence the proof. □

Theorem 2.12. *For $l \geq 5$,* $$\gamma_{p_3}((C(2l+1, 1, 2) = \begin{cases} 5, & l = 5 \\ 2\lfloor \frac{l}{2} \rfloor, & l \geq 6 \end{cases}.$$

Proof. Let $G = C(2l+1, 1, 2)$, $V(G)$ and $E(G)$ are as theorem 2.6.

For $l = 5, 6$ and 7, the sets $D_{p_3} = \{v_0, v_5, v_6, v_8, v_{10}\}$, $\{v_0, v_5, v_6, v_9, v_{10}, v_{12}\}$ and $\{v_0, v_5, v_6, v_9, v_{10}, v_{14}\}$ are minimal 3-path dsets and hence $\gamma_{p_3}(G)= 5, 6$ and 6 respectively.

Let $l \geq 8$.

Consider dset $D_{p_3} = \{v_0\} \cup \{v_{4j+1}\} \cup \{v_{4p+2}\} \cup \{v_{2l}\}$, where $1 \leq j, p \leq \lfloor \frac{l}{2} \rfloor - 1$. This dset D_{p_3} is a minimal 3-path dset since, for any $v \in D_{p_3}$, $D_{p_3} - \{v\}$ is n't a 3-path dset and also, few $u \in$

$V - D_{p3}$ is n't dominated by a bit of vertex in $D_{p_3} \cup \{v\}$. Hence, either u=v or $u \in V - D_{p_3}$. But, $degree(v) = 0$ of D_{p_3} if u=v, and, if $u \in V - D_{p_3}$ and u is n't dominated by $D_{p_3} - \{v\}$, but is dominated by D_{p_3}, then u is adjoining only to vertex v in D_{p_3}, i.e. $N(v) \cap D_{p_3}$ is equal to $\{v\}$. Therefore, $|D_{p_3}| = \begin{cases} 5, & l = 5 \\ 2\lfloor \frac{l}{2} \rfloor, & l \geq 6 \end{cases}$,
it follows that

$$\gamma_{p_3}(G) = \begin{cases} 5, & l = 5 \\ 2\lfloor \frac{l}{2} \rfloor, & l \geq 6 \end{cases}$$

Hence the proof.

□

Theorem 2.13. *For $l \geq 5$, $\gamma_{p_3}(C(2l+1, 1, 3)) = l$.*

Proof. Let $G = C(2l+1, 1, 3)$, $V(G)$ and $E(G)$ are as theorem 2.6. Consider the dset $D_{p_3} = \{v_0, v_2, v_4, v_6\} \cup \{v_{2j+7}\}$, where $1 \leq j \leq n-4$. This dset D_{p_3} is a minimal 3-path dset since, for any $v \in D_{p_3}$, $D_{p_3} - \{v\}$ is n't a 3-path dset and also, few $u \in V - D_{p_3}$ is n't dominated by a bit of vertex in $D_{p_3} \cup \{v\}$. Hence, either u=v or $u \in V - D_{p_3}$. But, $degree(v) = 0$ of D_{p_3} if u=v, and, if $u \in V - D_{p_3}$ and u is n't dominated by $D_{p_3} - \{v\}$, but is dominated by D_{p_3}, then u is adjoining only to vertex v in D_{p_3}, i.e., $N(v) \cap D_{p_3}$ is equal to $\{v\}$. Therefore, $|D_{p_3}| = l$, it follows that $\gamma_{p_3}(G) = l$.
Hence the proof.

□

Theorem 2.14. *For $l \geq 5$, $\gamma_{p_3}(C(2l+1, 1, 4)) = l+1$.*

Proof. Let $G = C(2l+1, 1, 4)$, $V(G)$ and $E(G)$ are as theorem 2.6. Consider the dset $D_{p_3} = \{v_0, v_2, v_3, v_5\} \cup \{v_{2j+4}\}$, where $1 \leq j \leq l-3$. This dset D_{p_3} is a minimal 3-path dset since, for any $v \in D_{p_3}$, $D_{p_3} - \{v\}$ is n't a 3-path dset and also, few $u \in V - D_{p_3}$ is n't dominated by a bit of vertex in $D_{p_3} \cup \{v\}$. Hence, either u=v or $u \in V - D_{p_3}$. But, $degree(v) = 0$ of D_{p_3} if u=v, and, if $u \in V - D_{p_3}$ and u is n't dominated by $D_{p_3} - \{v\}$, but is dominated by D_{p_3}, then u is adjoining only to vertex $v \in D_{p_3}$, i.e., $N(v) \cap D_{p_3}$ is equal to $\{v\}$. Therefore, $|D_{p_3}| = l+1$, it follows that $\gamma_{p_3}(G) = l+1$.
Hence the proof.

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