

## Intuitionistic Fuzzy $\beta^{**}G$ irresolute Mappings with Separation

### Axioms

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### Abstract

The aim of this paper is to introduce and study the concepts of theoretical applications of intuitionistic fuzzy  $\beta^{**}$  generalized closed sets by defining new spaces namely, intuitionistic fuzzy  $\beta^{**}$  generalized  $T_{1/2}$  space and intuitionistic fuzzy  $\beta^{**}$  pre  $T_{1/2}$  space. And also we have introduced intuitionistic fuzzy  $\beta^{**}$  generalized irresolute mappings and investigated some of their properties.

**Keywords:** Intuitionistic fuzzy topology, intuitionistic fuzzy  $\beta^{**}$  generalized closed set, intuitionistic fuzzy  $\beta^{**}$  generalized  $T_{1/2}$  space, intuitionistic fuzzy  $\beta^{**}$  pre  $T_{1/2}$  space and intuitionistic fuzzy  $\beta^{**}$  generalized irresolute mappings.

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### 1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] introduced intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Saranya and Jayanthi introduced intuitionistic fuzzy  $\beta$  generalized closed sets in 2016. Sudha and Jayanthi [6] introduced intuitionistic fuzzy  $\beta^{**}$  generalized closed sets. In this paper we overtly enunciate the notion of intuitionistic fuzzy  $\beta^{**}$  generalized  $T_{1/2}$  space, intuitionistic fuzzy  $\beta^{**}$  pre  $T_{1/2}$  space, intuitionistic fuzzy  $\beta^{**}$  generalized irresolute mappings and investigated some of their properties.

## 2. Preliminaries

**Definition 2.1:** [1] An **intuitionistic fuzzy set** (IFS)  $A$  is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions  $\mu_A : X \rightarrow [0, 1]$  and  $\nu_A : X \rightarrow [0, 1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$  respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ . Denote by  $\text{IFS}(X)$ , the set of all intuitionistic fuzzy sets in  $X$ . An intuitionistic fuzzy set  $A$  in  $X$  is simply denoted by  $A = \langle x, \mu_A, \nu_A \rangle$  instead of denoting  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ .

**Definition 2.2:** [1] Let  $A$  and  $B$  be two IFSs of the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$ . Then,

- (a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ ,
- (b)  $A = B$  if and only if  $A \subseteq B$  and  $A \supseteq B$ ,
- (c)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$ ,
- (d)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$ ,
- (e)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$ .

The intuitionistic fuzzy sets  $0_\sim = \langle x, 0, 1 \rangle$  and  $1_\sim = \langle x, 1, 0 \rangle$  are respectively the empty set and the whole set of  $X$ .

**Definition 2.3:** [2] An **intuitionistic fuzzy topology** (IFT) on  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms:

- (i)  $0_\sim, 1_\sim \in \tau$ ,
- (ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$
- (iii)  $\cup G_i \in \tau$  for any family  $\{ G_i : i \in J \} \subseteq \tau$

In this case the pair  $(X, \tau)$  is called the **intuitionistic fuzzy topological space** (IFTS) and any IFS in  $\tau$  is known as an **intuitionistic fuzzy open set** (IFOS) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS) in  $X$ .

**Definition 2.4:** [8] Two IFSs  $A$  and  $B$  are said to be  $q$ -coincident ( $A \text{ }_q \text{ } B$ ) if and only if there exists an element  $x \in X$  such that  $\mu_A(x) > \nu_B(x)$  or  $\nu_A(x) < \mu_B(x)$ .

**Definition 2.5:** [3] An **intuitionistic fuzzy point** (IFP), written as  $p_{(\alpha, \beta)}$  is defined to be an IFS of  $X$  given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP  $p_{(\alpha, \beta)}$  is said to belong to a set  $A$  if  $\alpha \leq \mu_A$  and  $\beta \geq \nu_A$ .

**Definition 2.6:** [6] An IFS  $A$  of an IFTS  $(X, \tau)$  is said to be an **intuitionistic fuzzy  $\beta^{**}$  generalized closed set** (IF $\beta^{**}$ GCS) if  $\text{cl}(\text{int}(\text{cl}(A))) \cap \text{int}(\text{cl}(\text{int}(A))) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $(X, \tau)$ .

The complement  $A^c$  of an IF $\beta^{**}$ GCS  $A$  in an IFTS  $(X, \tau)$  is called an **intuitionistic fuzzy  $\beta^{**}$  generalized open set** (IF $\beta^{**}$ GOS) in  $X$ .

The family of all IF $\beta^{**}$ GOSs of an IFTS  $(X, \tau)$  is denoted by IF $\beta^{**}$ GO( $X$ ).

**Result 2.7:** [6] Every IFCS, IFRCS, IFSCS, IFPCS, IF $\beta$ CS, IF $\alpha$ CS, IFGCS is an IF $\beta^{**}$ GCS but the converses may not true in general.

**Definition 2.8:** [9] An IFTS  $(X, \tau)$  is said to be an **intuitionistic fuzzy  $T_{1/2}$  space** if every IFGCS is an IFCS in  $(X, \tau)$ .

**Definition 2.9:** [4] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be an **intuitionistic fuzzy (IF) continuous** mapping if  $f^{-1}(B) \in \text{IFO}(X)$  for every  $B \in \sigma$ .

**Definition 2.10:** [7] A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called an **intuitionistic fuzzy  $\beta^{**}$  generalized continuous** (IF $\beta^{**}$ G continuous) **mapping** if  $f^{-1}(V)$  is an IF $\beta^{**}$ GCS in  $(X, \tau)$  for every IFCS  $V$  of  $(Y, \sigma)$ .

**Definition 2.11:** [5] A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called an **intuitionistic fuzzy generalized irresolute** (IF irresolute) **mapping** if  $f^{-1}(V)$  is an IFGCS in  $(X, \tau)$  for every IFGCS  $V$  of  $(Y, \sigma)$ .

### 3. Theoretical applications of intuitionistic fuzzy $\beta^{**}$ generalized closed sets

In this section we have investigated some theoretical applications of intuitionistic fuzzy  $\beta^{**}$  generalized closed sets by defining new spaces and obtained many interesting propositions.

**Definition 3.1:** An IFTS  $(X, \tau)$  is an *intuitionistic fuzzy  $\beta^{**}pT_{1/2}$ (IF $\beta^{**}pT_{1/2}$ ) space* if every IF $\beta^{**}$ GCS is an IFPCS in  $X$ .

**Example 3.2:** Let  $X = \{a, b\}$  and  $\tau = \{0_{\sim}, G_1, G_2, 1_{\sim}\}$  be an IFT on  $X$ , where  $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.4_b) \rangle$ ,  $G_2 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.3_b) \rangle$ .

Then,

IFPC(X) =  $\{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_a + \nu_a \leq 1\}$  and

IF $\beta^{**}$ GC(X) =  $\{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \mu_b \leq 1, 0 \leq \mu_a + \mu_b \leq 1\}$

Therefore the space  $(X, \tau)$  is an intuitionistic fuzzy  $\beta^{**}pT_{1/2}$  space, as every IF $\beta^{**}$ GCS is an IFPCS in this  $(X, \tau)$ .

**Definition 3.3:** An IFTS  $(X, \tau)$  is an *intuitionistic fuzzy  $\beta^{**}gT_{1/2}$ (IF $\beta^{**}gT_{1/2}$ ) space* if every IF $\beta^{**}$ GCS is an IFGCS in  $X$ .

**Example 3.4:** In example 3.2, IFGC(X) =  $\{0_{\sim}, 1_{\sim}, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_a + \nu_a \leq 1\}$ . The space  $(X, \tau)$  is an intuitionistic fuzzy  $\beta^{**}gT_{1/2}$  space, as every IF $\beta^{**}$ GCS is an IFGCS in this  $(X, \tau)$ .

**Remark 3.5:** Not every IF $\beta^{**}pT_{1/2}$  space is an IF $T_{1/2}$  space. This can be seen easily from the following example.

**Example 3.6:** In example 3.2,  $(X, \tau)$  is an IF $\beta^{**}pT_{1/2}$  space, but not an IF $T_{1/2}$  space. Since the IFS  $A = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b) \rangle$  is an IFGCS, but not an IFCS, as  $cl(A) = 1_{\sim} \neq A$ .

**Proposition 3.7:** An IFTS  $(X, \tau)$  is an  $IF\beta^{**}gT_{1/2}$  space if and only if  $IFGO(X) = IF\beta^{**}GO(X)$ .

**Proof: Necessity:** Let  $A$  be an  $IF\beta^{**}GOS$  in  $(X, \tau)$ , then  $A^c$  is an  $IF\beta^{**}GCS$  in  $(X, \tau)$ . By hypothesis,  $A^c$  is an  $IFGCS$  in  $(X, \tau)$ . Hence  $A$  is an  $IFGOS$  in  $(X, \tau)$ . Thus  $IFGO(X) = IF\beta^{**}GO(X)$ .

**Sufficiency:** Let  $A$  be an  $IF\beta^{**}GCS$  in  $(X, \tau)$ . Then  $A^c$  is an  $IF\beta^{**}GOS$  in  $(X, \tau)$ . By hypothesis,  $A^c$  is an  $IFGOS$  in  $(X, \tau)$ . Therefore  $A$  is an  $IFGCS$  in  $(X, \tau)$ . Hence  $(X, \tau)$  is an  $IF\beta^{**}gT_{1/2}$  space.

**Proposition 3.8:** Let  $X$  be an  $IF\beta^{**}pT_{1/2}$  space. Then for an IFS  $A$  the following conditions are equivalent:

- (i)  $A \in IF\beta^{**}GO(X)$
- (ii)  $A \subseteq \text{int}(\text{cl}(A))$
- (iii) There exists an IFOS  $G$  such that  $G \subseteq A \subseteq \text{int}(\text{cl}(A))$

**Proof:** (i)  $\Rightarrow$  (ii) Let  $A \in IF\beta^{**}GO(X)$ . This implies  $A$  is an IFPOS in  $X$ , since  $X$  is an  $IF\beta^{**}pT_{1/2}$  space. Then  $A^c$  is an IFPCS in  $X$ . Therefore  $\text{cl}(\text{int}(A^c)) \subseteq A^c$ . This implies  $A \subseteq \text{int}(\text{cl}(A))$ .

(ii)  $\Rightarrow$  (iii) Let  $A \subseteq \text{int}(\text{cl}(A))$ . Hence  $\text{int}(\text{int}(A)) \subseteq A \subseteq \text{int}(\text{cl}(A))$ . Then there exists IFOS  $G$  in  $X$  such that  $G \subseteq A \subseteq \text{int}(\text{cl}(A))$ , where  $G = \text{int}(A)$ .

(iii)  $\Rightarrow$  (i) Suppose that there exists IFOS  $G$  such that  $G \subseteq A \subseteq \text{int}(\text{cl}(A))$ . It is clear that  $(\text{int}(\text{cl}(A)))^c \subseteq A^c$ . This implies  $\text{cl}(\text{int}(A^c)) \subseteq A^c$ . That is  $A^c$  is an IFPCS in  $X$ . This implies  $A$  is an IFPOS in  $X$ . Hence  $A \in IF\beta^{**}GO(X)$ .

**Proposition 3.9:** Let  $X$  be an  $IF\beta^{**}gT_{1/2}$  space. If  $A$  is an IFS of  $X$  then the following properties are equivalent:

- (i)  $A \in IF\beta^{**}GO(X)$
- (ii)  $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $U \subseteq A$  and  $U$  is an IFCS in  $X$ .
- (iii) There exist IFOSs  $G$  and  $G_1$  such that  $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $A \in IF\beta^{**}GO(X)$ . This implies  $A$  is an IFGOS in  $X$ , since  $X$  is an  $IF\beta^{**}gT_{1/2}$  space. Then  $A^c$  is an IFGCS in  $X$ . Therefore  $\text{cl}(A^c) \subseteq V$  whenever  $A^c \subseteq V$  and  $V$  is an IFOS in  $X$ . That is  $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq \text{cl}(\text{cl}(A^c)) \subseteq \text{cl}(A^c) \subseteq V$ . This implies

$V^c \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $V^c \subseteq A$  and  $V^c$  is an IFCS in  $X$ . Replacing  $V^c$  by  $U$ ,  $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $U \subseteq A$  and  $U$  is an IFCS in  $X$ .

(ii)  $\Rightarrow$  (iii) Let  $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $U \subseteq A$  and  $U$  is an IFCS in  $X$ . We have  $\text{int}(U) \subseteq U \subseteq \text{int}(\text{cl}(\text{int}(A)))$ . Hence there exist IFOSs  $G$  and  $G_1$  in  $X$  such that  $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$  where  $G = \text{int}(A)$  and  $G_1 = \text{int}(U)$ .

(iii)  $\Rightarrow$  (i) Suppose that there exist IFOSs  $G$  and  $G_1$  such that  $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$ . Then it is clear that  $(\text{int}(\text{cl}(G)))^c \subseteq U^c$ . That is  $(\text{int}(\text{cl}(\text{int}(A))))^c \subseteq U^c$ . This implies  $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq U^c$ ,  $A^c \subseteq U^c$  and  $U^c$  is an IFOS in  $X$ . This implies  $\text{cl}(A^c) \subseteq U^c$ . That is  $A^c$  is an IFGCS in  $X$ . This implies  $A$  is an IFGOS in  $X$ . Hence  $A \in \text{IF}\beta^{**}\text{GO}(X)$ .

**Proposition 3.10:** Let  $(X, \tau)$  be an  $\text{IF}\beta^{**}\text{pT}_{1/2}$  space. Then

- (i) Any union of  $\text{IF}\beta^{**}\text{GCS}$  is an  $\text{IF}\beta^{**}\text{GCS}$ ,
- (ii) Any intersection of  $\text{IF}\beta^{**}\text{GOS}$  is an  $\text{IF}\beta^{**}\text{GOS}$ .

**Proof:** (i) Let  $\{A_i\}_{i \in I}$  be a collection of  $\text{IF}\beta^{**}\text{GCS}$ s. Since  $(X, \tau)$  is an  $\text{IF}\beta^{**}\text{pT}_{1/2}$  space, every  $\text{IF}\beta^{**}\text{GCS}$  is an IFPCS. As any union of IFPCS is an IFPCS,  $\bigcup_{i \in I} A_i$  is an IFPCS. Since every IFPCS is an  $\text{IF}\beta^{**}\text{GCS}$ ,  $\bigcup_{i \in I} A_i$  is an  $\text{IF}\beta^{**}\text{GCS}$ .

(ii) can be proved easily by taking complement in (i).

**Proposition 3.11:** An IFTS  $(X, \tau)$  is an intuitionistic fuzzy  $\beta^{**}\text{pT}_{1/2}$  space if and only if  $\text{IF}\beta^{**}\text{GO}(X) = \text{IFPO}(X)$ .

**Proof: Necessity:** Let  $A$  be an  $\text{IF}\beta^{**}\text{GOS}$  in  $(X, \tau)$ , then  $A^c$  is an  $\text{IF}\beta^{**}\text{GCS}$  in  $(X, \tau)$ . By hypothesis,  $A^c$  is an IFPCS in  $(X, \tau)$ . Hence  $A$  is an IFPOS in  $X$ . Thus  $\text{IF}\beta^{**}\text{GO}(X) = \text{IFPO}(X)$ .

**Sufficiency:** Let  $A$  be an  $\text{IF}\beta^{**}\text{GCS}$  in  $(X, \tau)$ . Then  $A^c$  is an  $\text{IF}\beta^{**}\text{GOS}$  in  $(X, \tau)$ . By hypothesis,  $A^c$  is an IFPOS in  $(X, \tau)$  and hence  $A$  is an IFPCS  $(X, \tau)$ . Therefore  $(X, \tau)$  is an intuitionistic fuzzy  $\beta^{**}\text{pT}_{1/2}$  space.

**Proposition 3.12:** For any IFS  $A$  in  $(X, \tau)$  where  $X$  is an  $IF\beta^{**}pT_{1/2}$  space,  $A \in IF\beta^{**}GO(X)$  if and only if for every IFP  $p_{(\alpha, \beta)} \in A$ , there exists an  $IF\beta^{**}GOS$   $B$  in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ .

**Proof: Necessity:** If  $A \in IF\beta^{**}GO(X)$ , then we can take  $B = A$  so that  $p_{(\alpha, \beta)} \in B \subseteq A$  for every IFP  $p_{(\alpha, \beta)} \in A$ .

**Sufficiency:** Let  $A$  be an IFS in  $(X, \tau)$  and assume that there exists  $B \in IF\beta^{**}GO(X)$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ . Since  $X$  is an  $IF\beta^{**}pT_{1/2}$  space,  $B$  is an IFPOS [6]. Then  $A = \bigcup_{p_{(\alpha, \beta)} \in A} \{p_{(\alpha, \beta)}\} \subseteq \bigcup_{p_{(\alpha, \beta)} \in A} B \subseteq A$ . Therefore  $A = \bigcup_{p_{(\alpha, \beta)} \in A} B$ , which is an IFPOS. Hence  $A$  is an  $IF\beta^{**}GOS$  in  $X$ .

#### 4. Intuitionistic fuzzy $\beta^{**}$ generalized irresolute mappings

In this section we have introduced intuitionistic fuzzy  $\beta^{**}$  generalized irresolute mappings and studied some of their properties.

**Definition 4.1:** A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an **intuitionistic fuzzy  $\beta^{**}$  generalized (IF $\beta^{**}G$ ) irresolutemapping** if  $f^{-1}(V)$  is an  $IF\beta^{**}GCS$  in  $(X, \tau)$  for every  $IF\beta^{**}GCS$   $V$  of  $(Y, \sigma)$ .

**Example 4.2:** Let  $X = \{a, b\}$  and  $Y = \{u, v\}$ . Then  $\tau = \{0_{\sim}, G_1, G_2, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_3, 1_{\sim}\}$  are IFTs on  $X$  and  $Y$  respectively, where  $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ ,  $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$  and  $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ . Define a mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an  $IF\beta^{**}G$  irresolute mapping.

**Proposition 4.3:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an  $IF\beta^{**}G$  irresolute mapping, then  $f$  is an  $IF\beta^{**}G$  continuous mapping but not conversely.

**Proof:** Let  $V$  be any IFCS in  $Y$ . Then  $V$  is an  $IF\beta^{**}GCS$  and by hypothesis  $f^{-1}(V)$  is an  $IF\beta^{**}GCS$  in  $X$ . Hence  $f$  is an  $IF\beta^{**}G$  continuous mapping.

**Example 4.4:** Let  $X = \{a, b\}$  and  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.6_a, 0.8_b), (0.2_a, 0.1_b) \rangle$ ,  $G_2 = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.2_b) \rangle$  and  $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$ . Then  $\tau = \{0_{\sim}, G_1, G_2, 1_{\sim}\}$  and

$\sigma = \{0_-, G_3, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ .

Then  $f$  is an  $IF\beta^{**}G$  continuous mapping but not an  $IF\beta^{**}G$  irresolute mapping, since the IFS  $A = \langle y, (0.5_u, 0.3_v), (0.2_u, 0.1_v) \rangle$  is an  $IF\beta^{**}GCS$  in  $Y$  and its inverse  $f^{-1}(A)$  is not an  $IF\beta^{**}GCS$  in  $X$ , as  $f^{-1}(A) = \langle x, (0.5_a, 0.3_b), (0.2_a, 0.1_b) \rangle \subseteq G_1$ , but  $\text{int}(\text{cl}(\text{int}(f^{-1}(A)))) \cap \text{cl}(\text{int}(\text{cl}(f^{-1}(A)))) = 1_-\not\subseteq G_1$ .

**Proposition 4.5:** A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an  $IF\beta^{**}G$  irresolute mapping if and only if the inverse image of each  $IF\beta^{**}GOS$  in  $Y$  is an  $IF\beta^{**}GOS$  in  $X$ .

**Proof :**straightforward.

**Proposition 4.6:**The composition of two  $IF\beta^{**}G$  irresolute mappings is an  $IF\beta^{**}G$  irresolute mapping.

**Proof :**Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  be any two  $IF\beta^{**}G$  irresolute mappings. Let  $V$  be an  $IF\beta^{**}GCS$  in  $Z$ . Then  $g^{-1}(V)$  is an  $IF\beta^{**}GCS$  in  $Y$ , by hypothesis. Since  $f$  is an  $IF\beta^{**}G$  irresolute mapping,  $f^{-1}(g^{-1}(V))$  is an  $IF\beta^{**}GCS$  in  $X$ . Hence  $g \circ f$  is an  $IF\beta^{**}G$  irresolute mapping.

**Proposition 4.7:**Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an  $IF\beta^{**}G$  irresolute mapping and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  is an  $IF\beta^{**}G$  continuous mapping, then  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an  $IF\beta^{**}G$  continuous mapping.

**Proof:**Let  $V$  be an IFCS in  $Z$ . Then  $g^{-1}(V)$  is an  $IF\beta^{**}GCS$  in  $Y$ . Since  $f$  is an  $IF\beta^{**}G$  irresolute mapping,  $f^{-1}(g^{-1}(V))$  is an  $IF\beta^{**}GCS$  in  $X$ . Hence  $g \circ f$  is an  $IF\beta^{**}G$  continuous mapping.

**Proposition 4.8:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a mapping from an IFTS  $X$  into an IFTS  $Y$ . Then the following conditions are equivalent if  $X$  and  $Y$  are  $IF\beta^{**}pT_{1/2}$  spaces:

- (i)  $f$  is an  $IF\beta^{**}G$  irresolute mapping
- (ii)  $f^{-1}(B)$  is an  $IF\beta^{**}GOS$  in  $X$  for each  $IF\beta^{**}GOS$   $B$  in  $Y$
- (iii)  $f^{-1}(\text{pint}(B)) \subseteq \text{pint}(f^{-1}(B))$  for each IFS  $B$  of  $Y$
- (iv)  $\text{pcl}(f^{-1}(B)) \subseteq f^{-1}(\text{pcl}(B))$  for each IFS  $B$  of  $Y$ .



**Proof:** (i)  $\Leftrightarrow$  (ii) is obvious, since  $f^{-1}(A^c) = (f^{-1}(A))^c$ .

(ii)  $\Rightarrow$  (iii) Let  $B$  be any IFS in  $Y$  and  $\text{pint}(B) \subseteq B$ . Also  $f^{-1}(\text{pint}(B)) \subseteq f^{-1}(B)$ . Since  $\text{pint}(B)$  is an IFPOS in  $Y$ , it is an  $\text{IF}\beta^{**}\text{GOS}$  in  $Y$ . Therefore  $f^{-1}(\text{pint}(B))$  is an  $\text{IF}\beta^{**}\text{GOS}$  in  $X$ , by hypothesis. Since  $X$  is an  $\text{IF}\beta^{**}\text{pT}_{1/2}$  space,  $f^{-1}(\text{pint}(B))$  is an IFPOS in  $X$ . Hence  $f^{-1}(\text{pint}(B)) = \text{pint}(f^{-1}(\text{pint}(B))) \subseteq \text{pint}(f^{-1}(B))$ .

(iii)  $\Rightarrow$  (iv) is obvious by taking complement in (iii).

(iv)  $\Rightarrow$  (i) Let  $B$  be an  $\text{IF}\beta^{**}\text{GCS}$  in  $Y$ . Since  $Y$  is an  $\text{IF}\beta^{**}\text{pT}_{1/2}$  space,  $B$  is an IFPCS in  $Y$  and  $\text{pcl}(B) = B$ . Hence  $f^{-1}(B) = f^{-1}(\text{pcl}(B)) \supseteq \text{pcl}(f^{-1}(B))$ , by hypothesis. But  $f^{-1}(B) \subseteq \text{pcl}(f^{-1}(B))$ . Therefore  $\text{pcl}(f^{-1}(B)) = f^{-1}(B)$ . This implies  $f^{-1}(B)$  is an IFPCS and hence it is an  $\text{IF}\beta^{**}\text{GCS}$  in  $X$ . Thus  $f$  is an  $\text{IF}\beta^{**}\text{G}$  irresolute mapping.

**Proposition 4.9:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an  $\text{IF}\beta^{**}\text{G}$  irresolute mapping. Then  $f^{-1}(B) \subseteq \text{pint}(f^{-1}(\text{int}(\text{cl}(B))))$  for every  $\text{IF}\beta^{**}\text{GOS}$   $B$  in  $Y$ , if  $X$  and  $Y$  are  $\text{IF}\beta^{**}\text{pT}_{1/2}$  spaces.

**Proof:** Let  $B$  be an  $\text{IF}\beta^{**}\text{GOS}$  in  $Y$ . Then by hypothesis,  $f^{-1}(B)$  is an  $\text{IF}\beta^{**}\text{GOS}$  in  $X$ . Since  $X$  is an  $\text{IF}\beta^{**}\text{pT}_{1/2}$  space,  $f^{-1}(B)$  is an IFPOS in  $X$ . Therefore  $\text{pint}(f^{-1}(B)) = f^{-1}(B)$ . Since  $Y$  is an  $\text{IF}\beta^{**}\text{pT}_{1/2}$  space,  $B$  is an IFPOS in  $Y$  and  $B \subseteq \text{int}(\text{cl}(B))$ . Now  $f^{-1}(B) = \text{pint}(f^{-1}(B)) \subseteq \text{pint}(f^{-1}(\text{int}(\text{cl}(B))))$ .

**Proposition 4.10:** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an  $\text{IF}\beta^{**}\text{G}$  irresolute mapping and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  is an IF contra continuous mapping, then  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an IF contra  $\beta^{**}\text{G}$  continuous mapping.

**Proof:** Let  $V$  be an IFOS in  $Z$ . Then  $g^{-1}(V)$  is an IFCS in  $Y$ , since  $g$  is an IF contra continuous mapping. As every IFCS is an  $\text{IF}\beta^{**}\text{GCS}$ ,  $g^{-1}(V)$  is an  $\text{IF}\beta^{**}\text{GCS}$  in  $Y$ . Since  $f$  is an  $\text{IF}\beta^{**}\text{G}$  irresolute mapping,  $f^{-1}(g^{-1}(V))$  is an  $\text{IF}\beta^{**}\text{GCS}$  in  $X$ . Therefore  $g \circ f$  is an IF contra  $\beta^{**}\text{G}$  continuous mapping.

**Proposition 4.11 :** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an  $\text{IF}\beta^{**}\text{G}$  irresolute mapping and  $g : (Y, \sigma) \rightarrow (Z, \delta)$  is an  $\text{IF}\beta^{**}\text{G}$  continuous mapping, then  $g \circ f : (X, \tau) \rightarrow (Z, \delta)$  is an IF almost  $\beta^{**}\text{G}$  continuous mapping.

**Proof :** Let  $V$  be an IFRCS in  $Z$ . Since every IFRCS is an IFCS,  $V$  is an IFCS in  $Z$ . Therefore  $g^{-1}(V)$  is an  $IF\beta^{**}GCS$  in  $Y$ , by hypothesis. Since  $f$  is an  $IF\beta^{**}G$  irresolute mapping,  $f^{-1}(g^{-1}(V))$  is an  $IF\beta^{**}GCS$  in  $X$ . Hence  $g \circ f$  is an IF almost  $\beta^{**}G$  continuous mapping.

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