Operation on Bipolar Spherical Fuzzy Graph

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Abstract: Spherical fuzzy set is more versatile than the existing fuzzy models, due to its outstanding feature of the vast space of uncertain and vagueness with the constraint $0 \leq \alpha^2 + \nu^2 + \beta^2 \leq 1$. Graph is a easy way to understand and handle a problem physically in the form of diagrams. In this paper, we discuss the operation on bipolar spherical fuzzy graphs namely, rejection with brief description on degree and total degree of bipolar spherical fuzzy graphs.

Keywords: spherical fuzzy set, bipolar spherical fuzzy graph, rejection.

I. INTRODUCTION

Zadeh [30] proposed fuzzy logic as an extension of classical logic which is a powerful proxy to probability theory to characterize uncertainty and vagueness in various fields. Atanassov[18] introduced intuitionistic fuzzy set by adding a non-membership function (β) to the membership function (α) of fuzzy set with the condition $0 \leq \alpha + \beta \leq 1$. Yager [29] extended Pythagorean fuzzy set from intuitionistic fuzzy set, which is characterized by the membership and non-membership function satisfying the condition that their square sum is not greater than 1. The concept of Pythagorean fuzzy number forwarded by Zhang and Xu [33]. Picture fuzzy set is a direct extension of intuitionistic fuzzy set initiated by Cuong [19,20] which gives three degrees to the elements namely, truthness (α), abstinence (ν) and falseness (β) under the limitation $0 \leq \alpha + \nu + \beta \leq 1$.

Gundogdu and Kahraman [23-25] introduced spherical fuzzy set as an extension of Pythagorean fuzzy set. The space of spherical fuzzy set membership degree is greater than the space of Pythagorean fuzzy set membership degree of truthness (α), abstinence (ν) and falseness (β) in the interval [0,1] with the limitation $0 \leq \alpha^2 + \nu^2 + \beta^2 \leq 1$. Ashraf et al [16,17] proposed the concept of spherical fuzzy sets, their operations and operators and defined MADM method to deal with spherical fuzzy information. Zhang [32] introduced bipolar fuzzy set, in which positive information represents the possiblity and negative information represents the impossibility. Lee [26] proposed the concept of bipolar valued fuzzy sets as a generalization of fuzzy sets, in which membership degree is enlarged to the interval [-1,1].

A graph is a convenient way of representing the data in which the objects are vertices and their relations are edges. Many real life problems can be solved by using graphs. Based on Zadeh's fuzzy relation [31], Kaufmann [22] introduced the idea of fuzzy graphs. Rosenfeld [27] studied the structure of fuzzy graphs by obtaining various fuzzy analogs such as cycles, paths and connectedness. Al-Hawary [12-14] considered certain concepts of fuzzy graphs. Parvathi and Karunambigai [21] extended the concept of intuitionistic fuzzy graph from the fuzzy graphs. Naz et al [27] introduced the idea of Pythagorean fuzzy graphs, an extension of intuitionistic fuzzy graphs. Akram et al [3-9] discussed the specific types of Pythagorean fuzzy graphs, including its applications in decision making. Akram et al [10] examined decision making methods based on spherical fuzzy graphs. Akram et al [11] extended the graph-theoretic concepts under a spherical fuzzy environment and discussed some operations, properties of irregular and edge-irregular spherical fuzzy graph with examples. The new idea of Bipolar neutrosophic cubic graphs and bipolar spherical fuzzy neutrosophic cubic graphs were studied in [1, 14] and some of their properties were discussed and also minimum spanning tree algorithm with numerical examples was presented. Recently, described symmetric difference operation on bipolar spherical fuzzy graph and also discussed some results to their degrees and total degrees with numerical example [2].

In this article, we study the operation rejection with a description on degree and total degree of bipolar spherical fuzzy graphs .

II. PRELIMINARIES

In this section, we study some basic definitions required to define rejection operation on bipolar spherical fuzzy graph.

Definition 2.1. Let A be an IFS in the universe of discourse X, shown as follows:

$$A = \{< x, \mu_A(x), \nu_A(x) >| x \in X\},$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and non-membership degree of element x belonging to the IFS A, respectively. Moreover, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitancy degree of element x belonging to the IFS A.

Definition 2.2. Let P be an PFS in the universe of discourse X, shown as follows:

$$P = \{< x, \mu_A(x), \nu_A(x) >| x \in X\},$$

where $\mu_P(x) : X \to [0,1]$ and $\nu_P(x) : X \to [0,1]$ satisfy $0 \le (\mu_A(x))^2 + (\nu_A(x))^2 \le 1$ for all $x \in X$, $\mu_p(x)$ and $\nu_P(x)$ denote the membership degree and non-membership degree of element x belonging to the PFS P, respectively. Moreover, $\pi_P(x) = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ is called the hesitancy degree of element *x* belonging to the PFS P. For convenience, we introduce a Pythagorean fuzzy number denoted by $\beta = P(\mu_\beta, \nu_\beta)$, where $\mu_\beta, \nu_\beta \in [0,1]$ and $0 \le (\mu_\beta)^2 + (\nu_\beta)^2 \le 1$.

Definition 2.3. Bipolar Neutrosophic Set

A bipolar neutrosophic set A in X is defined as an object of the form

$$P = \{< x, T_P^+(x), I_P^+(x), F_P^+(x), T_P^-(x), I_P^-(x), F_P^-(x) >| x \in X\},$$

where $T_P^+, I_P^+, F_P^+ : X \to [0,1]$, $T_P^-, I_P^-, F_P^- : X \to [0,1]$.

Definition 2.4. Spherical Fuzzy Set

Let *X* be a universe. Then the set

$$P = \{< x, (T_P(x), I_P(x), F_P(x)) >| x \in X\},$$

is said to be spherical fuzzy set, where $T_P(x): X \to [0,1]$, $I_P(x): X \to [0,1]$ and $F_P(x): X \to [0,1]$ are said to be degree of positive-membership function of x in X, degree of neutral-membership function of x in X and degree of negative-membership function of x in X, respectively. Also T_P, I_P and F_P satisfy the following condition:

$$(\forall x \in X) \quad (0 \le (T_P(x))^2 + (I_P(x))^2 + (F_P(x))^2 \le 1).$$

Definition 2.5

Let X be a non-empty set. A Bipolar Spherical Fuzzy Set (BSFS)

$$A = \left\{ < x, T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N > | \, x \in X \right\}$$

where $T_A^P, I_A^P, F_A^P : X \to [0,1]$, $T_A^N, I_A^N, F_A^N : X \to [-1,0]$ are the mappings such that $0 \le \left(\left(T_A^P\right)^2 + \left(I_A^P\right)^2 + \left(F_A^P\right)^2 \right) \le 1$ and $0 \le \left(\left(T_A^N\right)^2 + \left(I_A^N\right)^2 + \left(F_A^N\right)^2 \right) \le 1$ and T_A^P denote the positive truth membership function, I_A^P denote the positive indeterminacy membership function, F_A^P denote the positive falsity membership function, T_A^N denote the negative truth membership function, I_A^N denote the negative indeterminacy membership function, F_A^N denote the negative falsity membership function.

Definition 2.6

A Bipolar Spherical Fuzzy Graph (BSFG) on an underlying set V is a pair $G = (A, B)$ where A is a bipolar spherical fuzzy set in V and B is a bipolar spherical fuzzy relation on $V \times V$ such that

$T_B^P(x, y) \le \min(T_A^P(x), T_A^P(y)), \qquad T_B^N(x, y) \ge \max(T_A^N(x), T_A^N(y))$

$I_B^P(x, y) \le \min(I_A^P(x), I_A^P(y)), \qquad I_B^N(x, y) \ge \max(I_A^N(x), I_A^N(y))$

$F_B^P(x, y) \le \max(F_A^P(x), F_A^P(y)), \qquad F_B^N(x, y) \ge \min(F_A^N(x), F_A^N(y))$

where T_B^P denote the positive truth membership function, I_B^P denote the positive indeterminacy membership function, F_B^P denote the positive falsity membership function, T_B^N denote the negative truth membership function, I_B^N denote the negative indeterminacy membership function, F_B^N denote the negative falsity membership function and fulfils the following conditions: $0 \le \left(\left(T_B^P\right)^2 + \left(I_B^P\right)^2 + \left(F_B^P\right)^2 \right) \le 1$ and $0 \le \left(\left(T_B^N\right)^2 + \left(I_B^N\right)^2 + \left(F_B^N\right)^2 \right) \le 1$, where A is a bipolar spherical fuzzy vertex set and B is a bipolar spherical fuzzy edge set of G.

Definition 2.7

Let $G = (A, B)$ be an BSFG defined on $G^* = (V, E)$. The order of BSFG is defined by

$$O(G) = \sum_{a \in b} \{T_A^P(x), I_A^P(x), F_A^P(x), T_A^N(x), I_A^N(x), F_A^N(x)\}$$

and the degree of a vertex x of G is defined by

$$\deg(x) = \sum_{xy \in E} \{T_B^P(xy), I_B^P(xy), F_B^P(xy), T_B^N(xy), I_B^N(xy), F_B^N(xy)\}.$$

Definition 2.8

Let $G=(A,B)$ be an BSFG defined on $G^*=(V,E)$. The total degree of a vertex x of G is defined by

$$t\deg(x)=\sum_{xy\in E}\begin{Bmatrix} T_B^P(xy)+T_A^P(x), I_B^P(xy)+I_A^P(x), F_B^P(xy)+F_A^P(x), \\ T_B^N(xy)+T_A^N(x), I_B^N(xy)+I_A^N(x), F_B^N(xy)+F_A^N(x)\end{Bmatrix}.$$

III. BIPOLAR SPHERICAL FUZZY GRAPH

In this section, we define rejection operation on bipolar spherical fuzzy graph and discuss some results.

Definition 3.1

Let $A_1=(T_{A_1}^P, I_{A_1}^P, F_{A_1}^P, T_{A_1}^N, I_{A_1}^N, F_{A_1}^N)$ and $A_2=(T_{A_2}^P, I_{A_2}^P, F_{A_2}^P, T_{A_2}^N, I_{A_2}^N, F_{A_2}^N)$ be bipolar spherical fuzzy sets defined on V_1 and V_2, and let $B_1=(T_{B_1}^P, I_{B_1}^P, F_{B_1}^P, T_{B_1}^N, I_{B_1}^N, F_{B_1}^N)$ and $B_2=(T_{B_2}^P, I_{B_2}^P, F_{B_2}^P, T_{B_2}^N, I_{B_2}^N, F_{B_2}^N)$ be bipolar spherical fuzzy sets defined on E_1 and E_2, respectively. Then, we denote the rejection of two BSFGs G_1 and G_2 of the graphs G_1^* and G_2^* by $G_1 | G_2=(A_1 | A_2, B_1 | B_2)$ and define as follows:

1. $\forall (x_1,x_2)\in V\times V$

$$(T_{A_1}^P | T_{A_2}^P)(x_1,x_2)=\min(T_{A_1}^P(x_1), T_{A_2}^P(x_2))$$
$$(I_{A_1}^P | I_{A_2}^P)(x_1,x_2)=\min(I_{A_1}^P(x_1), I_{A_2}^P(x_2))$$
$$(F_{A_1}^P | F_{A_2}^P)(x_1,x_2)=\max(F_{A_1}^P(x_1), F_{A_2}^P(x_2))$$
$$(T_{A_1}^N | T_{A_2}^N)(x_1,x_2)=\max(T_{A_1}^N(x_1), T_{A_2}^N(x_2))$$
$$(I_{A_1}^N | I_{A_2}^N)(x_1,x_2)=\max(I_{A_1}^N(x_1), I_{A_2}^N(x_2))$$
$$(F_{A_1}^N | F_{A_2}^N)(x_1,x_2)=\min(F_{A_1}^N(x_1), F_{A_2}^N(x_2))$$

2. $\forall x\in V_1$ and $x_2y_2\in E_2$

$$(T_{B_1}^P | T_{B_2}^P)((x,x_2)(x,y_2))=\min(T_{A_1}^P(x), T_{A_2}^P(x_2), T_{A_2}^P(y_2))$$
$$(I_{B_1}^P | I_{B_2}^P)((x,x_2)(x,y_2))=\min(I_{A_1}^P(x), I_{A_2}^P(x_2), I_{A_2}^P(y_2))$$
$$(F_{B_1}^P | F_{B_2}^P)((x,x_2)(x,y_2))=\max(F_{A_1}^P(x), F_{A_2}^P(x_2), F_{A_2}^P(y_2))$$
$$(T_{B_1}^N | T_{B_2}^N)((x,x_2)(x,y_2))=\max(T_{A_1}^N(x), T_{A_2}^N(x_2), T_{A_2}^N(y_2))$$
$$(I_{B_1}^N | I_{B_2}^N)((x,x_2)(x,y_2))=\max(I_{A_1}^N(x), I_{A_2}^N(x_2), I_{A_2}^N(y_2))$$
$$(F_{B_1}^N | F_{B_2}^N)((x,x_2)(x,y_2))=\min(F_{A_1}^N(x), F_{A_2}^N(x_2), F_{A_2}^N(y_2))$$

3. $\forall z\in V_2$ and $x_1y_1\notin E_1$

$$(T_{B_1}^P | T_{B_2}^P)((x_1,z)(y_1,z))=\min(T_{B_1}^P(x_1), T_{B_1}^P(y_1), T_{A_2}^P(z))$$
$$(I_{B_1}^P | I_{B_2}^P)((x_1,z)(y_1,z))=\min(I_{B_1}^P(x_1), I_{B_1}^P(y_1), I_{A_2}^P(z))$$
$$(F_{B_1}^P | F_{B_2}^P)((x_1,z)(y_1,z))=\max(F_{B_1}^P(x_1), F_{B_1}^P(y_1), F_{A_2}^P(z))$$

$$(T^N_{B_1} | T^N_{B_2})((x_1, z)(y_1, z)) = \max(T^N_{B_1}(x_1), T^N_{B_1}(y_1), T^N_{A_2}(z))$$
$$(I^N_{B_1} | I^N_{B_2})((x_1, z)(y_1, z)) = \max(I^N_{B_1}(x_1), I^N_{B_1}(y_1), I^N_{A_2}(z))$$
$$(F^N_{B_1} | F^N_{B_2})((x_1, z)(y_1, z)) = \min(F^N_{B_1}(x_1), F^N_{B_1}(y_1), F^N_{A_2}(z))$$

4. $\forall\, x_1y_1 \notin E_1, x_2y_2 \notin E_2$

$$(T^P_{B_1} | T^P_{B_2})((x_1, x_2)(y_1, y_2)) = \min(T^P_{A_1}(x_1), T^P_{A_1}(y_1), T^P_{A_2}(x_2), T^P_{A_2}(y_2))$$
$$(I^P_{B_1} | I^P_{B_2})((x_1, x_2)(y_1, y_2)) = \min(I^P_{A_1}(x_1), I^P_{A_1}(y_1), I^P_{A_2}(x_2), I^P_{A_2}(y_2))$$
$$(F^P_{B_1} | F^P_{B_2})((x_1, x_2)(y_1, y_2)) = \max(F^P_{A_1}(x_1), F^P_{A_1}(y_1), F^P_{A_2}(x_2), F^P_{A_2}(y_2))$$

$$(T^N_{B_1} | T^N_{B_2})((x_1, x_2)(y_1, y_2)) = \max(T^N_{A_1}(x_1), T^N_{A_1}(y_1), T^N_{A_2}(x_2), T^N_{A_2}(y_2))$$
$$(I^N_{B_1} | I^N_{B_2})((x_1, x_2)(y_1, y_2)) = \max(I^N_{A_1}(x_1), I^N_{A_1}(y_1), I^N_{A_2}(x_2), I^N_{A_2}(y_2))$$
$$(F^N_{B_1} | F^N_{B_2})((x_1, x_2)(y_1, y_2)) = \min(F^N_{A_1}(x_1), F^N_{A_1}(y_1), F^N_{A_2}(x_2), F^N_{A_2}(y_2))$$

Proposition 3.2

If G_1 and G_2 are BSFGs, then $G_1 | G_2$ is a BSFG.

Proof

Let $x \in V_1$ and $x_2y_2 \notin E_2$. Then we have

$$\begin{aligned}(T^P_{B_1} | T^P_{B_2})((x, x_2)(x, y_2)) &= \min\{T^P_{A_1}(x), T^P_{B_2}(x_2y_2)\}\\
&\le \min\{T^P_{A_1}(x), \min(T^P_{A_2}(x_2), T^P_{A_2}(y_2))\}\\
&= \min\{\min(T^P_{A_1}(x), T^P_{A_2}(x_2)), \min(T^P_{A_1}(x), T^P_{A_2}(y_2))\}\\
&= \min\{((T^P_{A_1} | T^P_{A_2})(x, x_2)), ((T^P_{A_1} | T^P_{A_2})(x, y_2))\}\end{aligned}$$

$$\begin{aligned}(T^N_{B_1} | T^N_{B_2})((x, x_2)(x, y_2)) &= \max\{T^N_{A_1}(x), T^N_{B_2}(x_2y_2)\}\\
&\le \max\{T^N_{A_1}(x), \max(T^N_{A_2}(x_2), T^N_{A_2}(y_2))\}\\
&= \max\{\max(T^N_{A_1}(x), T^N_{A_2}(x_2)), \max(T^N_{A_1}(x), T^N_{A_2}(y_2))\}\\
&= \max\{((T^N_{A_1} | T^N_{A_2})(x, x_2)), ((T^N_{A_1} | T^N_{A_2})(x, y_2))\}\end{aligned}$$

$$\begin{aligned}(I^P_{B_1} | I^P_{B_2})((x, x_2)(x, y_2)) &= \min\{I^P_{A_1}(x), I^P_{B_2}(x_2y_2)\}\\
&\le \min\{I^P_{A_1}(x), \min(I^P_{A_2}(x_2), I^P_{A_2}(y_2))\}\\
&= \min\{\min(I^P_{A_1}(x), I^P_{A_2}(x_2)), \min(I^P_{A_1}(x), I^P_{A_2}(y_2))\}\\
&= \min\{((I^P_{A_1} | I^P_{A_2})(x, x_2)), ((I^P_{A_1} | I^P_{A_2})(x, y_2))\}\end{aligned}$$

$$\begin{aligned}(I^N_{B_1} | I^N_{B_2})((x, x_2)(x, y_2)) &= \max\{I^N_{A_1}(x), I^N_{B_2}(x_2y_2)\}\\
&\le \max\{I^N_{A_1}(x), \max(I^N_{A_2}(x_2), I^N_{A_2}(y_2))\}\\
&= \max\{\max(I^N_{A_1}(x), I^N_{A_2}(x_2)), \max(I^N_{A_1}(x), I^N_{A_2}(y_2))\}\\
&= \max\{((I^N_{A_1} | I^N_{A_2})(x, x_2)), ((I^N_{A_1} | I^N_{A_2})(x, y_2))\}\end{aligned}$$

$$
\begin{aligned}
(F_{B_1}^P \mid F_{B_2}^P)((x,x_2)(x,y_2)) &= \max\{F_{A_1}^P(x), F_{B_2}^P(x_2y_2)\} \\
&\le \max\{F_{A_1}^P(x), \max(F_{A_2}^P(x_2), F_{A_2}^P(y_2))\} \\
&= \max\{\max(F_{A_1}^P(x), F_{A_2}^P(x_2)), \max(F_{A_1}^P(x), F_{A_2}^P(y_2))\} \\
&= \max\{((F_{A_1}^P \mid F_{A_2}^P)(x,x_2)), ((F_{A_1}^P \mid F_{A_2}^P)(x,y_2))\}
\end{aligned}
$$

$$
\begin{aligned}
(F_{B_1}^N \mid F_{B_2}^N)((x,x_2)(x,y_2)) &= \min\{F_{A_1}^N(x), F_{B_2}^N(x_2y_2)\} \\
&\le \min\{F_{A_1}^N(x), \min(F_{A_2}^N(x_2), F_{A_2}^N(y_2))\} \\
&= \min\{\min(F_{A_1}^N(x), F_{A_2}^N(x_2)), \min(F_{A_1}^N(x), F_{A_2}^N(y_2))\} \\
&= \min\{((F_{A_1}^N \mid F_{A_2}^N)(x,x_2)), ((F_{A_1}^N \mid F_{A_2}^N)(x,y_2))\}
\end{aligned}
$$

Let $z \in V_2$ and $x_1y_1 \notin E_1$. Then we have

$$
\begin{aligned}
(T_{B_1}^P \mid T_{B_2}^P)((x_1,z)(y_1,z)) &= \min\{T_{B_1}^P(x_1y_1), T_{A_2}^P(z)\} \\
&\le \min\{\min(T_{A_1}^P(x_1), T_{A_1}^P(y_1)), T_{A_2}^P(z)\} \\
&= \min\{\min(T_{A_1}^P(x_1), T_{A_2}^P(z)), \min(T_{A_1}^P(y_1), T_{A_2}^P(z))\} \\
&= \min\{((T_{A_1}^P \mid T_{A_2}^P)(x_1,z)), ((T_{A_1}^P \mid T_{A_2}^P)(y_1,z))\}
\end{aligned}
$$

$$
\begin{aligned}
(T_{B_1}^N \mid T_{B_2}^N)((x_1,z)(y_1,z)) &= \max\{T_{B_1}^N(x_1y_1), T_{A_2}^N(z)\} \\
&\le \max\{\max(T_{A_1}^N(x_1), T_{A_1}^N(y_1)), T_{A_2}^N(z)\} \\
&= \max\{\max(T_{A_1}^N(x_1), T_{A_2}^N(z)), \max(T_{A_1}^N(y_1), T_{A_2}^N(z))\} \\
&= \max\{((T_{A_1}^N \mid T_{A_2}^N)(x_1,z)), ((T_{A_1}^N \mid T_{A_2}^N)(y_1,z))\}
\end{aligned}
$$

$$
\begin{aligned}
(I_{B_1}^P \mid I_{B_2}^P)((x_1,z)(y_1,z)) &= \min\{I_{B_1}^P(x_1y_1), I_{A_2}^P(z)\} \\
&\le \min\{\min(I_{A_1}^P(x_1), I_{A_1}^P(y_1)), I_{A_2}^P(z)\} \\
&= \min\{\min(I_{A_1}^P(x_1), I_{A_2}^P(z)), \min(I_{A_1}^P(y_1), I_{A_2}^P(z))\} \\
&= \min\{((I_{A_1}^P \mid I_{A_2}^P)(x_1,z)), ((I_{A_1}^P \mid I_{A_2}^P)(y_1,z))\}
\end{aligned}
$$

$$
\begin{aligned}
(I_{B_1}^N \mid I_{B_2}^N)((x_1,z)(y_1,z)) &= \max\{I_{B_1}^N(x_1y_1), I_{A_2}^N(z)\} \\
&\le \max\{\max(I_{A_1}^N(x_1), I_{A_1}^N(y_1)), I_{A_2}^N(z)\} \\
&= \max\{\max(I_{A_1}^N(x_1), I_{A_2}^N(z)), \max(I_{A_1}^N(y_1), I_{A_2}^N(z))\} \\
&= \max\{((I_{A_1}^N \mid I_{A_2}^N)(x_1,z)), ((I_{A_1}^N \mid I_{A_2}^N)(y_1,z))\}
\end{aligned}
$$

$$
\begin{aligned}
(F_{B_1}^P \mid F_{B_2}^P)((x_1,z)(y_1,z)) &= \max\{F_{B_1}^P(x_1y_1), F_{A_2}^P(z)\} \\
&\le \max\{\max(F_{A_1}^P(x_1), F_{A_1}^P(y_1)), F_{A_2}^P(z)\} \\
&= \max\{\max(F_{A_1}^P(x_1), F_{A_2}^P(z)), \max(F_{A_1}^P(y_1), F_{A_2}^P(z))\} \\
&= \max\{((F_{A_1}^P \mid F_{A_2}^P)(x_1,z)), ((F_{A_1}^P \mid F_{A_2}^P)(y_1,z))\}
\end{aligned}
$$

$$
\begin{aligned}
(F^N_{B_1} \mid F^N_{B_2})((x_1,z)(y_1,z)) &= \min\{F^N_{B_1}(x_1y_1), F^N_{A_2}(z)\} \\
&\le \min\{\min(F^N_{A_1}(x_1), F^N_{A_1}(y_1)), F^N_{A_2}(z)\} \\
&= \min\{\min(F^N_{A_1}(x_1), F^N_{A_2}(z)), \min(F^N_{A_1}(y_1), F^N_{A_2}(z))\} \\
&= \min\{((F^N_{A_1} \mid F^N_{A_2})(x_1,z)), ((F^N_{A_1} \mid F^N_{A_2})(y_1,z))\}
\end{aligned}
$$

Let $x_1y_1 \notin E_1, x_2y_2 \notin E_2$. Then we have

$$
\begin{aligned}
(T^P_{B_1} \mid T^P_{B_2})((x_1,x_2)(y_1,y_2)) &= \min\{T^P_{A_1}(x_1), T^P_{A_1}(y_1), T^P_{B_2}(x_2y_2)\} \\
&\le \min\{T^P_{A_1}(x_1), T^P_{A_1}(y_1), \min(T^P_{A_2}(x_2), T^P_{A_2}(y_2))\} \\
&= \min\{\min(T^P_{A_1}(x_1), T^P_{A_2}(x_2)), \min(T^P_{A_1}(y_1), T^P_{A_2}(y_2))\} \\
&= \min\{((T^P_{A_1} \mid T^P_{A_2})(x_1,x_2)), ((T^P_{A_1} \mid T^P_{A_2})(y_1,y_2))\}
\end{aligned}
$$

$$
\begin{aligned}
(T^N_{B_1} \mid T^N_{B_2})((x_1,x_2)(y_1,y_2)) &= \min\{T^N_{A_1}(x_1), T^N_{A_1}(y_1), T^N_{B_2}(x_2y_2)\} \\
&\le \min\{T^N_{A_1}(x_1), T^N_{A_1}(y_1), \min(T^N_{A_2}(x_2), T^N_{A_2}(y_2))\} \\
&= \min\{\min(T^N_{A_1}(x_1), T^N_{A_2}(x_2)), \min(T^N_{A_1}(y_1), T^N_{A_2}(y_2))\} \\
&= \min\{((T^N_{A_1} \mid T^N_{A_2})(x_1,x_2)), ((T^N_{A_1} \mid T^N_{A_2})(y_1,y_2))\}
\end{aligned}
$$

$$
\begin{aligned}
(I^P_{B_1} \mid I^P_{B_2})((x_1,x_2)(y_1,y_2)) &= \min\{I^P_{A_1}(x_1), I^P_{A_1}(y_1), I^P_{B_2}(x_2y_2)\} \\
&\le \min\{I^P_{A_1}(x_1), I^P_{A_1}(y_1), \min(I^P_{A_2}(x_2), I^P_{A_2}(y_2))\} \\
&= \min\{\min(I^P_{A_1}(x_1), I^P_{A_2}(x_2)), \min(I^P_{A_1}(y_1), I^P_{A_2}(y_2))\} \\
&= \min\{((I^P_{A_1} \mid I^P_{A_2})(x_1,x_2)), ((I^P_{A_1} \mid I^P_{A_2})(y_1,y_2))\}
\end{aligned}
$$

$$
\begin{aligned}
(I^N_{B_1} \mid I^N_{B_2})((x_1,x_2)(y_1,y_2)) &= \max\{I^N_{A_1}(x_1), I^N_{A_1}(y_1), I^N_{B_2}(x_2y_2)\} \\
&\le \max\{I^N_{A_1}(x_1), I^N_{A_1}(y_1), \max(I^N_{A_2}(x_2), I^N_{A_2}(y_2))\} \\
&= \max\{\max(I^N_{A_1}(x_1), I^N_{A_2}(x_2)), \max(I^N_{A_1}(y_1), I^N_{A_2}(y_2))\} \\
&= \max\{((I^N_{A_1} \mid I^N_{A_2})(x_1,x_2)), ((I^N_{A_1} \mid I^N_{A_2})(y_1,y_2))\}
\end{aligned}
$$

$$
\begin{aligned}
(F^P_{B_1} \mid F^P_{B_2})((x_1,x_2)(y_1,y_2)) &= \max\{F^P_{A_1}(x_1), F^P_{A_1}(y_1), F^P_{B_2}(x_2y_2)\} \\
&\le \max\{F^P_{A_1}(x_1), F^P_{A_1}(y_1), \max(F^P_{A_2}(x_2), F^P_{A_2}(y_2))\} \\
&= \max\{\max(F^P_{A_1}(x_1), F^P_{A_2}(x_2)), \max(F^P_{A_1}(y_1), F^P_{A_2}(y_2))\} \\
&= \max\{((F^P_{A_1} \mid F^P_{A_2})(x_1,x_2)), ((F^P_{A_1} \mid F^P_{A_2})(y_1,y_2))\}
\end{aligned}
$$

$$
\begin{aligned}
(F^N_{B_1} \mid F^N_{B_2})((x_1,x_2)(y_1,y_2)) &= \min\{F^N_{A_1}(x_1), F^N_{A_1}(y_1), F^N_{B_2}(x_2y_2)\} \\
&\le \min\{F^N_{A_1}(x_1), F^N_{A_1}(y_1), \min(F^N_{A_2}(x_2), F^N_{A_2}(y_2))\} \\
&= \min\{\min(F^N_{A_1}(x_1), F^N_{A_2}(x_2)), \min(F^N_{A_1}(y_1), F^N_{A_2}(y_2))\} \\
&= \min\{((F^N_{A_1} \mid F^N_{A_2})(x_1,x_2)), ((F^N_{A_1} \mid F^N_{A_2})(y_1,y_2))\}
\end{aligned}
$$

Hence $G_1 \mid G_2$ is an BSFG.

Definition 3.3

Let $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ be two BSFGs. Then for any vertex, $(x_1,x_2)\in V_1\times V_2$.

$$\begin{aligned}
\deg(T^P_{G_1}\mid T^P_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(T^P_{B_1}\mid T^P_{B_2})(x_1,x_2)(y_1,y_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\min\{T^P_{A_1}(x_1),T^P_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\min\{T^P_{A_2}(x_2),T^P_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\min\{T^P_{B_1}(x_1y_1),T^P_{B_2}(x_2y_2)\}
\end{aligned}$$

$$\begin{aligned}
\deg(T^N_{G_1}\mid T^N_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(T^N_{B_1}\mid T^N_{B_2})(x_1,x_2)(y_1,y_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\max\{T^N_{A_1}(x_1),T^N_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\max\{T^N_{A_2}(x_2),T^N_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\max\{T^N_{B_1}(x_1y_1),T^N_{B_2}(x_2y_2)\}
\end{aligned}$$

$$\begin{aligned}
\deg(I^P_{G_1}\mid I^P_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(I^P_{B_1}\mid I^P_{B_2})(x_1,x_2)(y_1,y_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\min\{I^P_{A_1}(x_1),I^P_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\min\{I^P_{A_2}(x_2),I^P_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\min\{I^P_{B_1}(x_1y_1),I^P_{B_2}(x_2y_2)\}
\end{aligned}$$

$$\begin{aligned}
\deg(I^N_{G_1}\mid I^N_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(I^N_{B_1}\mid I^N_{B_2})(x_1,x_2)(y_1,y_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\max\{I^N_{A_1}(x_1),I^N_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\max\{I^N_{A_2}(x_2),I^N_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\max\{I^N_{B_1}(x_1y_1),I^N_{B_2}(x_2y_2)\}
\end{aligned}$$

$$\begin{aligned}
\deg(F^P_{G_1}\mid F^P_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(F^P_{B_1}\mid F^P_{B_2})(x_1,x_2)(y_1,y_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\max\{F^P_{A_1}(x_1),F^P_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\max\{F^P_{A_2}(x_2),F^P_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\max\{F^P_{B_1}(x_1y_1),F^P_{B_2}(x_2y_2)\}
\end{aligned}$$

$$\begin{aligned}
\deg(F^N_{G_1}\mid F^N_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(F^N_{B_1}\mid F^N_{B_2})(x_1,x_2)(y_1,y_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\min\{F^N_{A_1}(x_1),F^N_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\min\{F^N_{A_2}(x_2),F^N_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\min\{F^N_{B_1}(x_1y_1),F^N_{B_2}(x_2y_2)\}
\end{aligned}$$

Definition 3.4

Let $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ be two BSFGs. Then for any vertex, $(x_1,x_2)\in V_1\times V_2$.

$$\begin{aligned}
t\deg(T^P_{G_1}\mid T^P_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2}(T^P_{B_1}\mid T^P_{B_2})(x_1,x_2)(y_1,y_2)+(T^P_{A_1}\mid T^P_{A_2})(x_1,x_2)\\
&= \sum_{x_1=y_1,x_2y_2\notin E_2}\min\{T^P_{A_1}(x_1),T^P_{B_2}(x_2y_2)\}+\sum_{x_2=y_2,x_1y_1\notin E_1}\min\{T^P_{A_2}(x_2),T^P_{B_1}(x_1y_1)\}\\
&\quad+\sum_{x_1y_1\notin E_1,x_2y_2\notin E_2}\min\{T^P_{B_1}(x_1y_1),T^P_{B_2}(x_2y_2)\}+\min\{(T^P_{A_1}(x_1),T^P_{A_2}(x_2)\}
\end{aligned}$$

$$\begin{aligned} t\deg(T^N_{G_1} \mid T^N_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2} (T^N_{B_1} \mid T^N_{B_2})(x_1,x_2)(y_1,y_2) + (T^N_{A_1} \mid T^N_{A_2})(x_1,x_2) \\ &= \sum_{x_1=y_1,x_2y_2\notin E_2} \max\{T^N_{A_1}(x_1), T^N_{B_2}(x_2y_2)\} + \sum_{x_2=y_2,x_1y_1\notin E_1} \max\{T^N_{A_2}(x_2), T^N_{B_1}(x_1y_1)\} \\ &\quad + \sum_{x_1y_1\notin E_1,x_2y_2\notin E_2} \max\{T^N_{B_1}(x_1y_1), T^N_{B_2}(x_2y_2)\} + \max\{(T^N_{A_1}(x_1), T^N_{A_2}(x_2)\} \end{aligned}$$

$$\begin{aligned} t\deg(I^P_{G_1} \mid I^P_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2} (I^P_{B_1} \mid I^P_{B_2})(x_1,x_2)(y_1,y_2) + (I^P_{A_1} \mid I^P_{A_2})(x_1,x_2) \\ &= \sum_{x_1=y_1,x_2y_2\notin E_2} \min\{I^P_{A_1}(x_1), I^P_{B_2}(x_2y_2)\} + \sum_{x_2=y_2,x_1y_1\notin E_1} \min\{I^P_{A_2}(x_2), I^P_{B_1}(x_1y_1)\} \\ &\quad + \sum_{x_1y_1\notin E_1,x_2y_2\notin E_2} \min\{I^P_{B_1}(x_1y_1), I^P_{B_2}(x_2y_2)\} + \min\{(I^P_{A_1}(x_1), I^P_{A_2}(x_2)\} \end{aligned}$$

$$\begin{aligned} t\deg(I^N_{G_1} \mid I^N_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2} (I^N_{B_1} \mid I^N_{B_2})(x_1,x_2)(y_1,y_2) + (I^N_{A_1} \mid I^N_{A_2})(x_1,x_2) \\ &= \sum_{x_1=y_1,x_2y_2\notin E_2} \max\{I^N_{A_1}(x_1), I^N_{B_2}(x_2y_2)\} + \sum_{x_2=y_2,x_1y_1\notin E_1} \max\{I^N_{A_2}(x_2), I^N_{B_1}(x_1y_1)\} \\ &\quad + \sum_{x_1y_1\notin E_1,x_2y_2\notin E_2} \max\{I^N_{B_1}(x_1y_1), I^N_{B_2}(x_2y_2)\} + \max\{(I^N_{A_1}(x_1), I^N_{A_2}(x_2)\} \end{aligned}$$

$$\begin{aligned} t\deg(F^P_{G_1} \mid F^P_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2} (F^P_{B_1} \mid F^P_{B_2})(x_1,x_2)(y_1,y_2) + (F^P_{A_1} \mid F^P_{A_2})(x_1,x_2) \\ &= \sum_{x_1=y_1,x_2y_2\notin E_2} \max\{F^P_{A_1}(x_1), F^P_{B_2}(x_2y_2)\} + \sum_{x_2=y_2,x_1y_1\notin E_1} \max\{F^P_{A_2}(x_2), F^P_{B_1}(x_1y_1)\} \\ &\quad + \sum_{x_1y_1\notin E_1,x_2y_2\notin E_2} \max\{F^P_{B_1}(x_1y_1), F^P_{B_2}(x_2y_2)\} + \max\{(F^P_{A_1}(x_1), F^P_{A_2}(x_2)\} \end{aligned}$$

$$\begin{aligned} t\deg(F^N_{G_1} \mid F^N_{G_2})(x_1,x_2) &= \sum_{(x_1,x_2)(y_1,y_2)\in E_1|E_2} (F^N_{B_1} \mid F^N_{B_2})(x_1,x_2)(y_1,y_2) + (F^N_{A_1} \mid F^N_{A_2})(x_1,x_2) \\ &= \sum_{x_1=y_1,x_2y_2\notin E_2} \min\{F^N_{A_1}(x_1), F^N_{B_2}(x_2y_2)\} + \sum_{x_2=y_2,x_1y_1\notin E_1} \min\{F^N_{A_2}(x_2), F^N_{B_1}(x_1y_1)\} \\ &\quad + \sum_{x_1y_1\notin E_1,x_2y_2\notin E_2} \min\{F^N_{B_1}(x_1y_1), F^N_{B_2}(x_2y_2)\} + \min\{(F^N_{A_1}(x_1), F^N_{A_2}(x_2)\} \end{aligned}$$

IV. CONCLUSION

Spherical fuzzy set is an extension of picture fuzzy set and Pythagorean fuzzy set. There is a need of spherical fuzzy set to tackle an interesting scenario emerge when existing sets failed to handle. The concept of bipolar fuzzy sets is a generalization of fuzzy set to deal with uncertainty and vagueness. Graph theory ideas are widely used to study various applications in different fields. In this article, we have discussed rejection operation on bipolar spherical fuzzy graph and developed results related to their degrees and total degrees. In future, we plan to extend this study to different areas where there are factors of decision making exists.

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